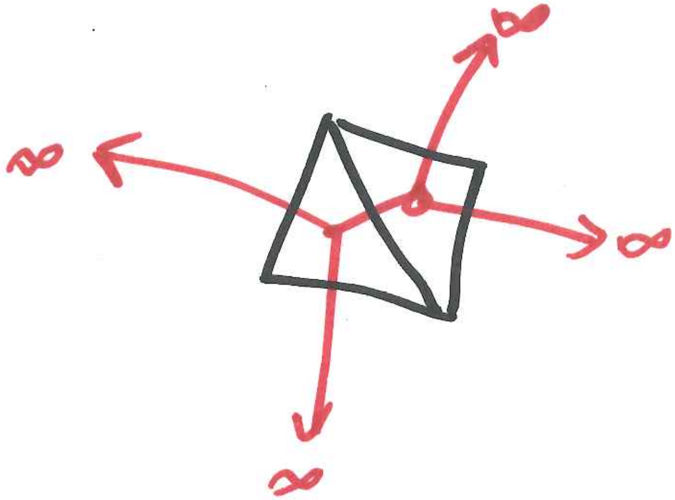
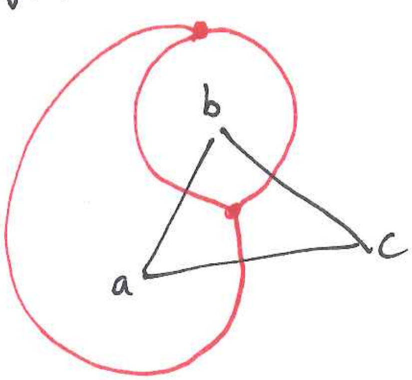
A Convenience: Put the dual vertex to the outer face "at infinity."

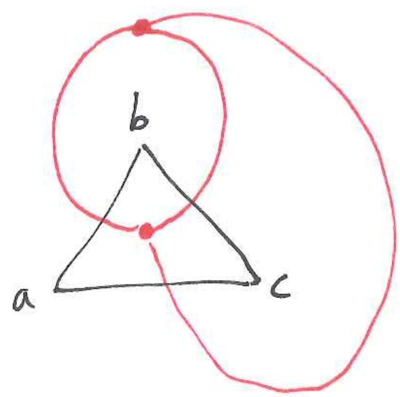


So, the underlying space is not $\mathbb{R}^2$, but rather $\mathbb{R}^2 \cup \{\infty\}$

It's the sphere S_2

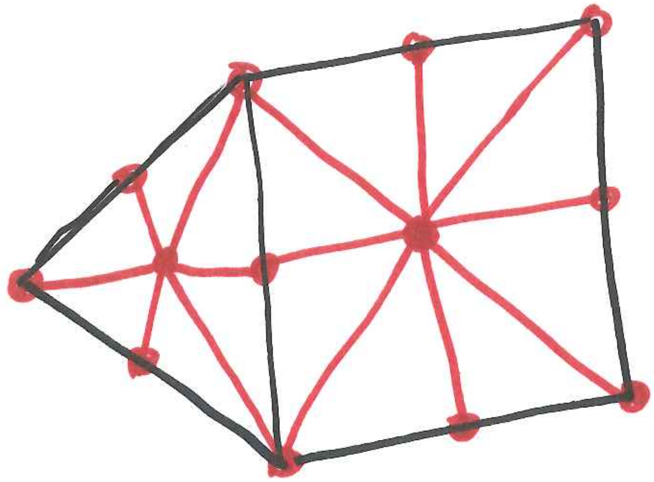
Which face is the infinite face in the dual?





It could be any vertex of the CH.

Barycentric Decomposition

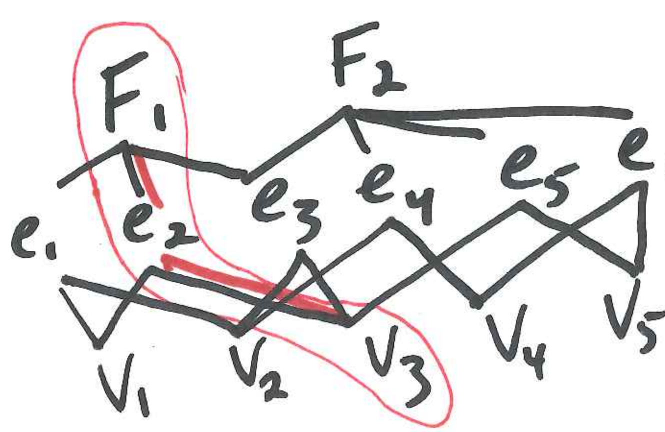


A vertex for each cell (V, E, or F), a triangle for each $(v,e,f) \in V \times E \times F$ s.t. $v < e < f$.

For now, we assume 2-connectivity so that the barycentric decomposition is a <u>triangulation</u> (i.e. all faces are triangles).

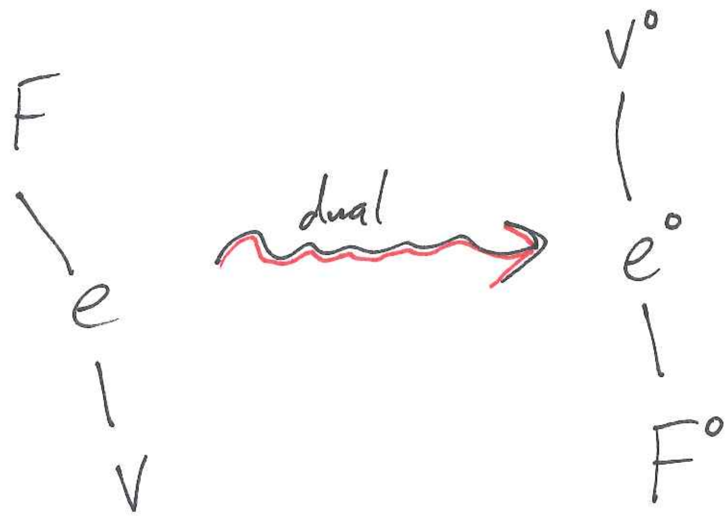
ex) Not 2-connected

In the Poset, A barycentric triangle is a maximal chain.

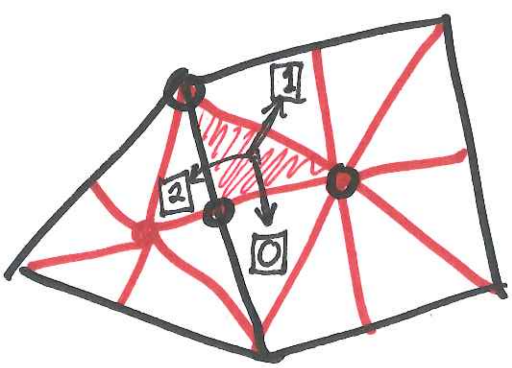


The barycentric decomposition encodes both primal and dual.

(because it encodes the poset)



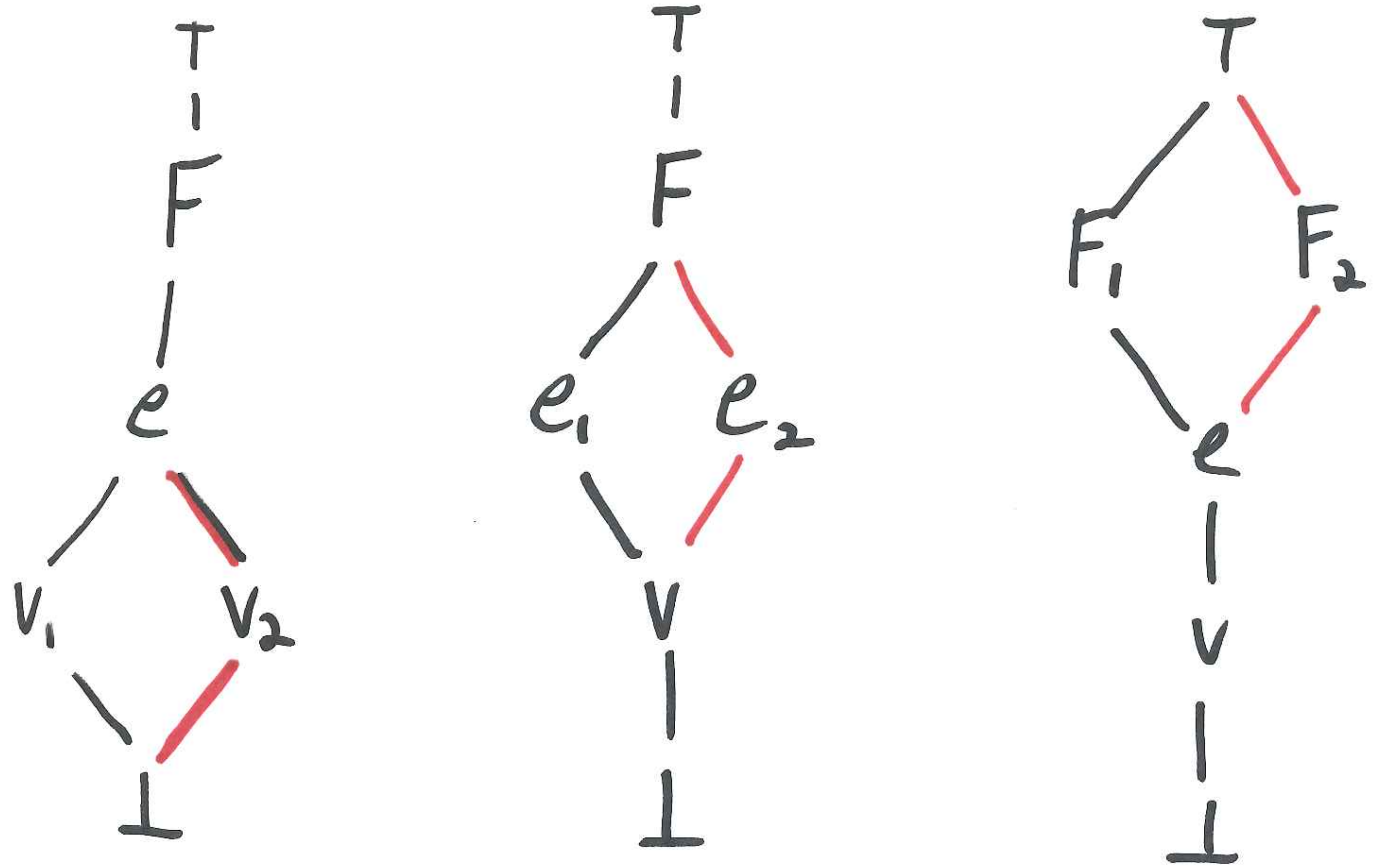
We can navigate the BD



handles are triangles

3 moves
- change vertex $\boxed{0}$
- change edge $\boxed{1}$
- change face $\boxed{2}$

How do these operations relate to the doubly connected edge list?



0 1 2

Diamonds in the Hasse Dgm indicate adjacency in the BD.

Polyhedral Complex

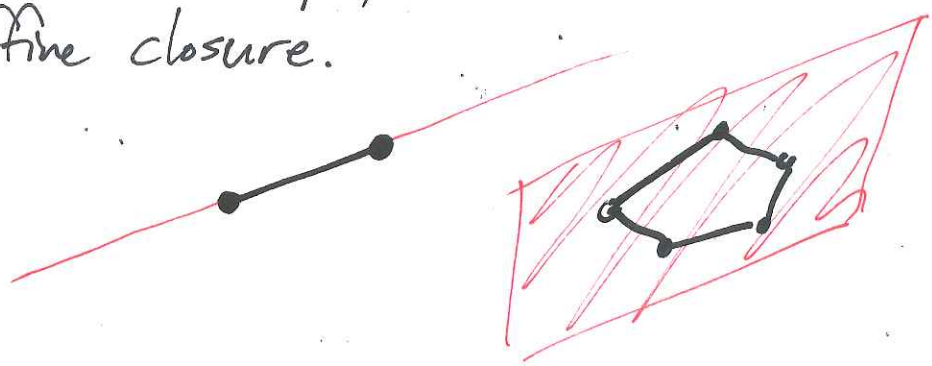
<u>Def</u> A <u>polyhedron</u> is the intersection of a finite set of closed halfspaces.

Caveat: Could be unbounded.

ex) In $\mathbb{R}^2$ it's a polygon (or a line segment).

ex) Cubes in $\mathbb{R}^3$

<u>Def</u> the <u>dimension</u> of a polyhedron is the dimension of its affine closure.



Some facts

~~Def~~ The boundary of a polyhedron is the union of polyhedra of one dimension lower.

So a polyhedron also has a poset.

<u>Def</u> A polyhedron contained in the boundary of polyhedron P is called a <u>face</u> of the polyhedron P.

A polyhedral complex is a set of polyhedra that is closed under taking boundaries and for any pair of polyhedra P_1, P_2

$$P_1 \cap P_2 = \left\{ \emptyset \quad \text{or} \quad \begin{array}{l} \text{a common face} \\ \text{of } P_1, P_2. \end{array} \right\}$$