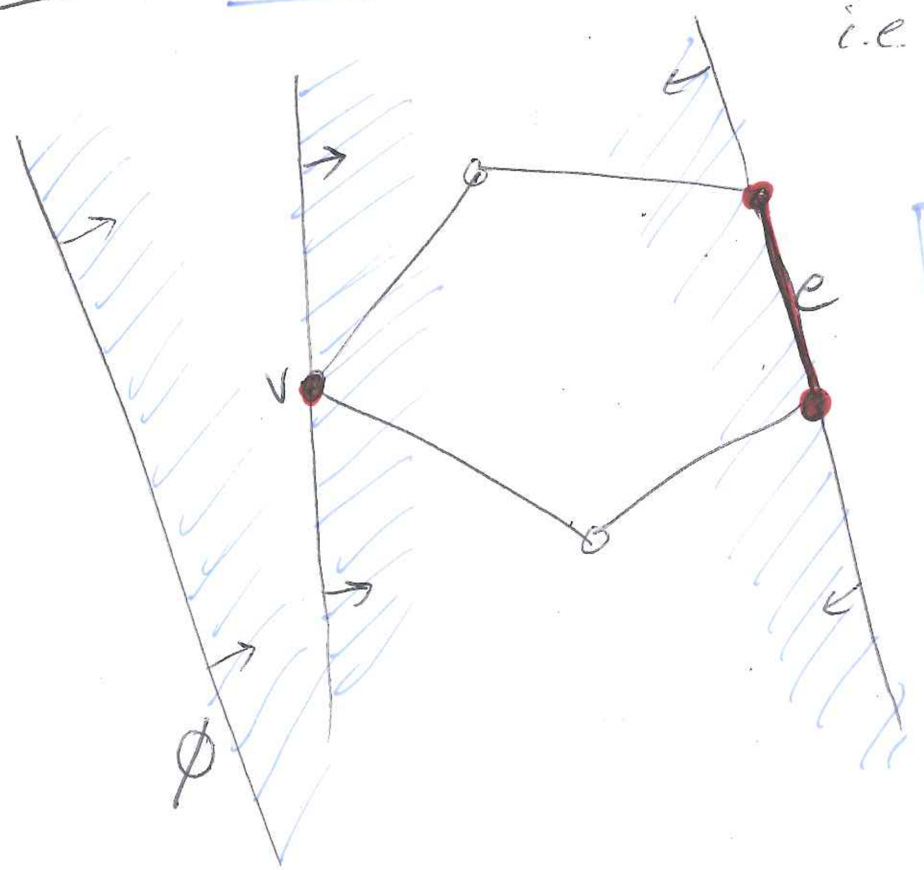
Def For a convex set S, a supporting hyperplane H is one that has all of S in one closed halfspace bounded by H.

Def A face of a polyhedron P is the intersection of P with a supporting hyperplane. We also say (by convention) that P is a face of itself.

Def A facet is a face of codimension 1.
i.e. dimension $= d-1$.



Note: The polyhedral complex gives a poset

Def For a polyhedral complex K, the underlying space $|K| = \bigcup_{P \in K} P$.

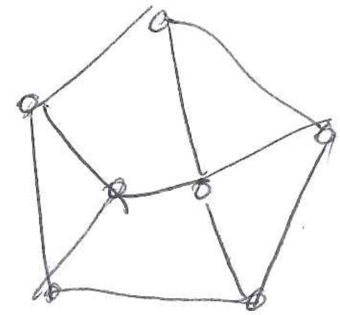
Def We'll say K is a decomposition of S if $S = |K|$.

Convex
Decomposition from a point set

Input: $P \subset \mathbb{R}^2$

Output: Polyhedral Complex K s.t.
$P =$ 0-faces of K and
$|K| = CC(P)$.

Convex closure

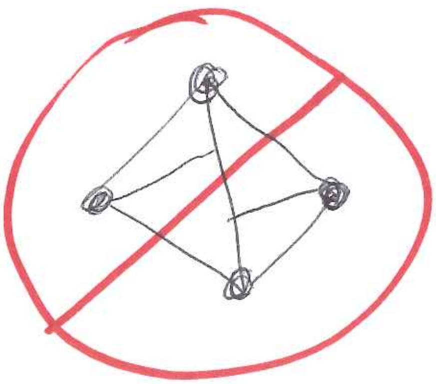


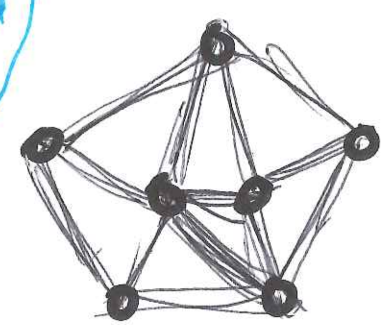
We say a convex decomposition of P is a triangulation of P if every 2-face is a triangle.

Sometimes denoted Δ^n

Note: for $P \subset \mathbb{R}^d$, we say K is a Δ^n if every d-face is a simplex.

convex closure of $d+1$ affinely independent points



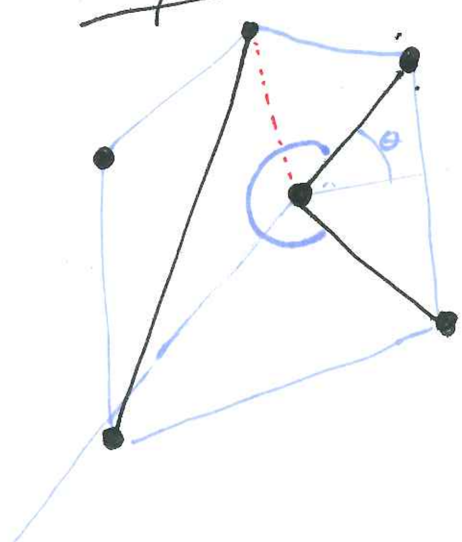


Triangulations Exist

A Paper and pen algorithm for triangulating a point set:

> If you can draw a straight edge between two vertices that doesn't cross any other edges,
> Then draw it.
> Repeat.

Why does this give a Δ^n?



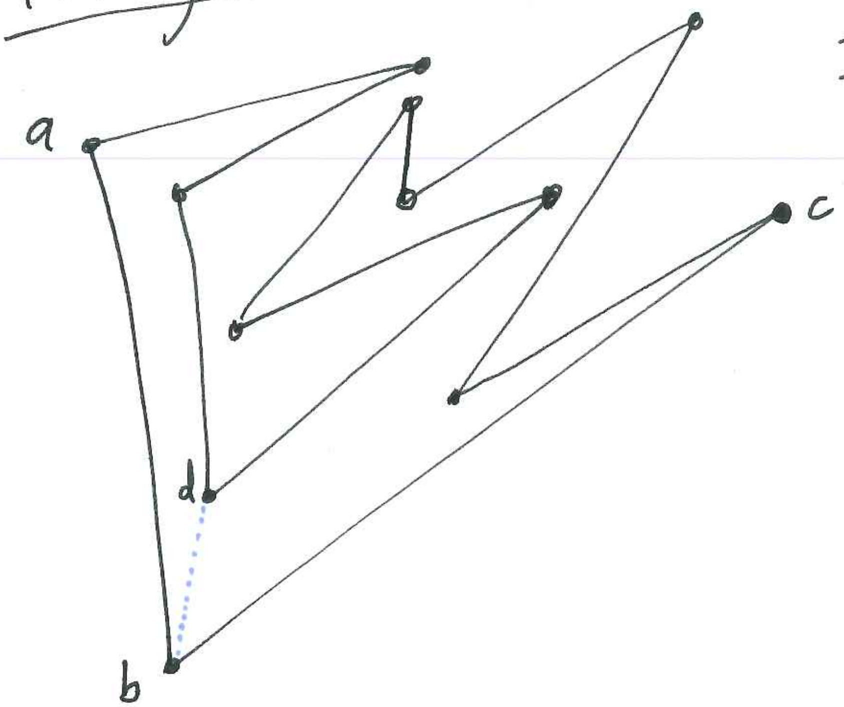
First, observe that the C.H. is in the output.

Next, for each interior vertex and angle θ there must be an edge in the range of directions $[\theta, \theta + \pi]$.

So, all faces are convex polygons.

Last, convex polygons can be triangulated.

Triangulations of Simple Polygons Exist



Idea: pick any non-reflex angle abc.
If Δabc is empty, add edge $\overline{ac}$.
If Δabc nonempty, add edge $\overline{ad}$ where d is the vertex in Δabc closest to edge $\overline{ab}$