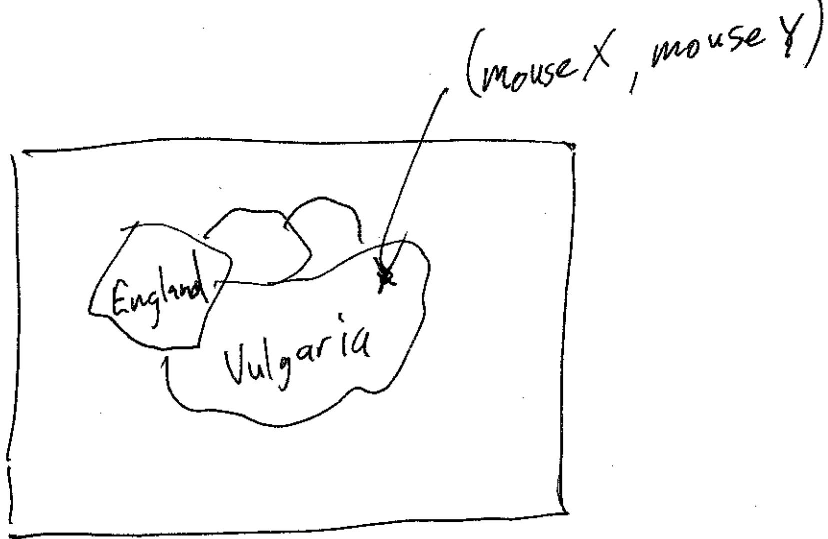


Problem: planar subdivision search

Def A polygonal subdivision is a cell complex where all cells are polygons.

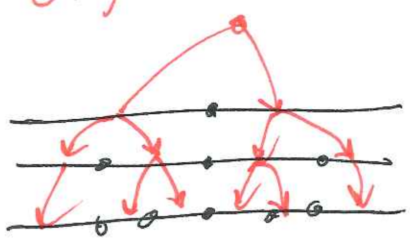
$n ::= \#$ polygons

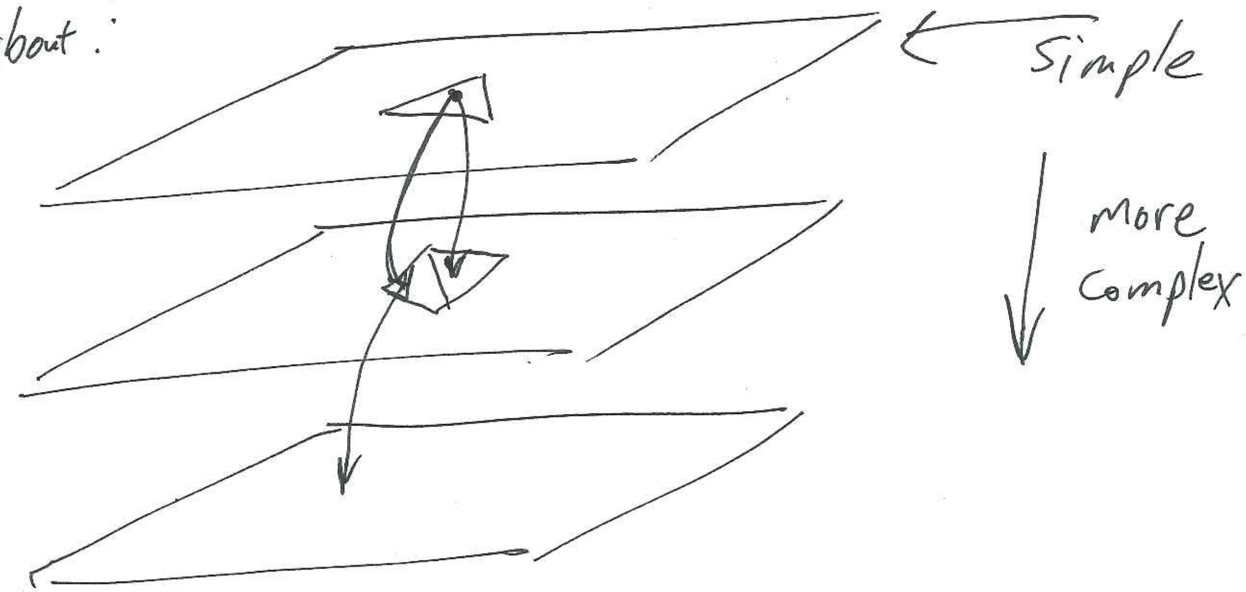
WLOG $|V| = O(n)$

We care about:

$O(1)$ → 1) Degree

$O(\log n)$ → 2) Depth





[Kirkpatrick's Algorithm '81]

randomized incremental Delaunay triangulation.

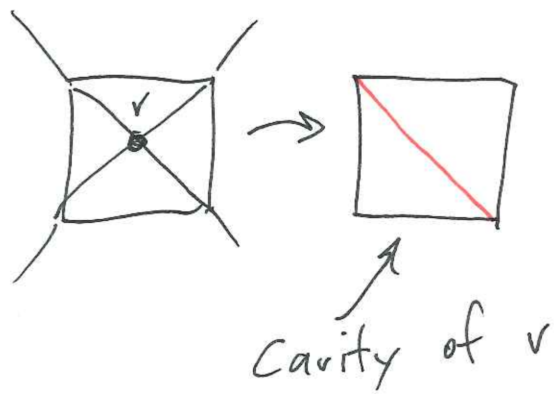
How is this different from R.I.D?

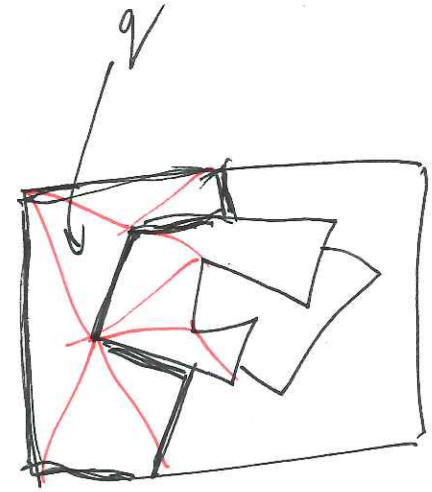
1) We don't know the queries in advance.

2) We care about <u>worst-case</u>.

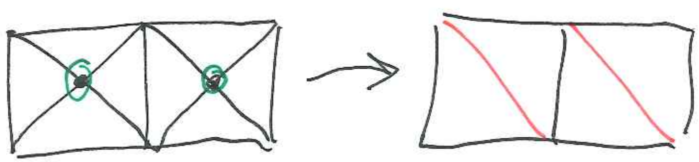
Idea 1 Think Backwards

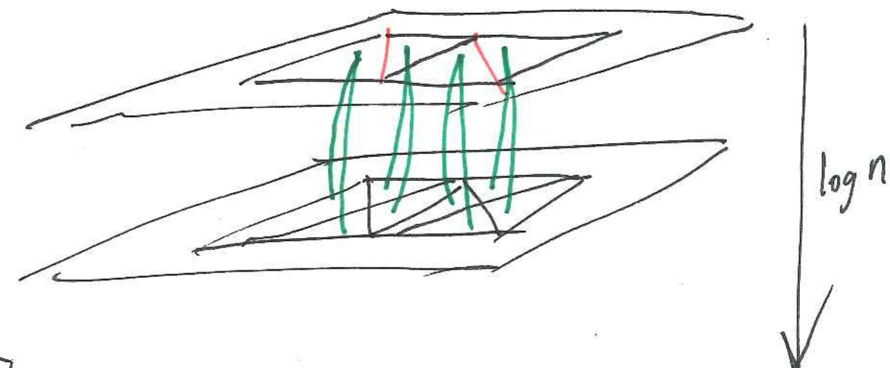
Idea 2 Restrict ourselves to Δns



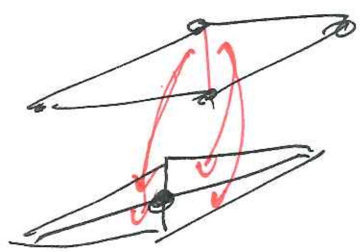


Idea 3 Think Parallel





Obs: If $u \not\to v$ then $\text{cavity}(u) \cap \text{cavity}(v) = \emptyset$



Obs: Degree in DAG determined by the degree of the vertex remove.

Lem In plane Δ^n, At least $\frac{|V|}{2}$ have deg < 12.

pf Suppose for contr. that $\frac{|V|}{2}$ vertices have deg ≥ 12

$$|E| = \frac{1}{2}\sum_{v \in V} \deg(v) \geq \frac{1}{2}\left(\frac{|V|}{2} 12\right) = 3|V| > |E|.$$

choose

<u>Lem</u>: In a plane Δ^n we can $\frac{|V|}{24}$ ~~[illegible]~~ ind. vertices of deg < 12, in $O(n)$ time.

<u>pf</u> Greedy Alg

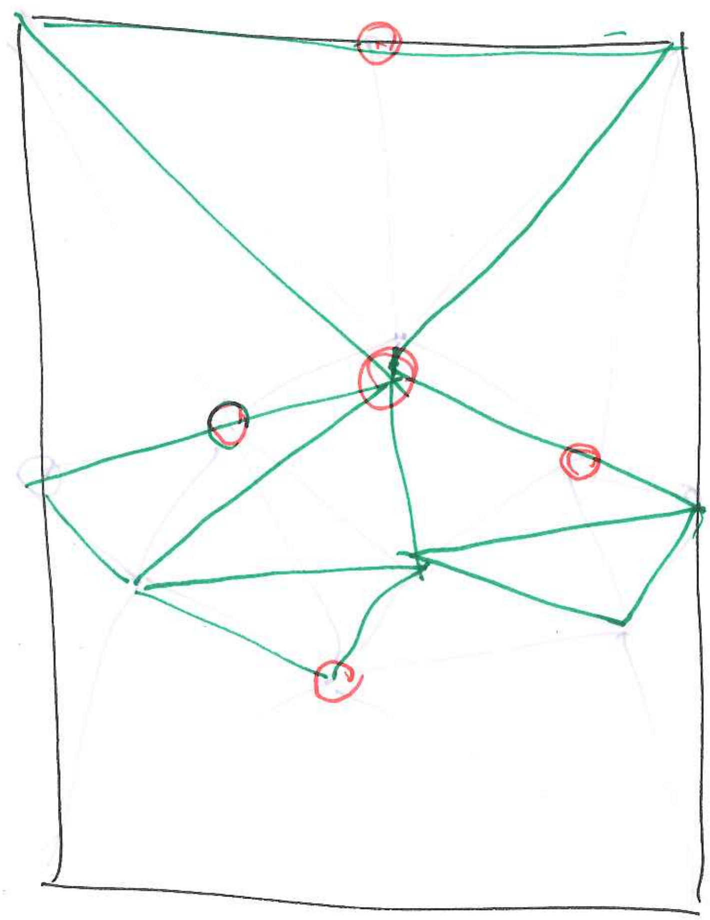
Repeatedly

pick low deg verts

throw out neighbors

$\frac{|V|}{2}$ low deg vertices

Finish w/ $\geq \frac{|V|}{24}$.



<u>Thm</u> A query takes $O(\log n)$ time.

<u>pf</u>

$n_i = \#$ vertices at level i

$n_1 = |V|$

$n_{i+1} = \left(\frac{23}{24}\right) n_i$

Depth d s.t. $n_d = O(1)$

$O(1) = n_d \leq \left(\frac{23}{24}\right)^d |V|$

$d = \log_{\frac{24}{23}} |V| = O(\log n)$

Thm On a Δ^n input. Preprocessing Time is ~~$O(n \log n)$~~ $O(n)$

pf At each level

- Find IS $[O(n)]$
- Retriangulating $[O(n)]$
- Checking HDAG edges $[O(n)]$

$O(\log n)$ levels. $\boxed{O(n)}$

Thm Space is $O(n)$

$T(n) = T\left(\frac{23}{24} n\right) + O(n)$

$$c \sum_{i=1}^{O(\log_{24/23} n)} \left(\frac{24}{23}\right)^i = O$$

$\left(\frac{24}{23}\right)$