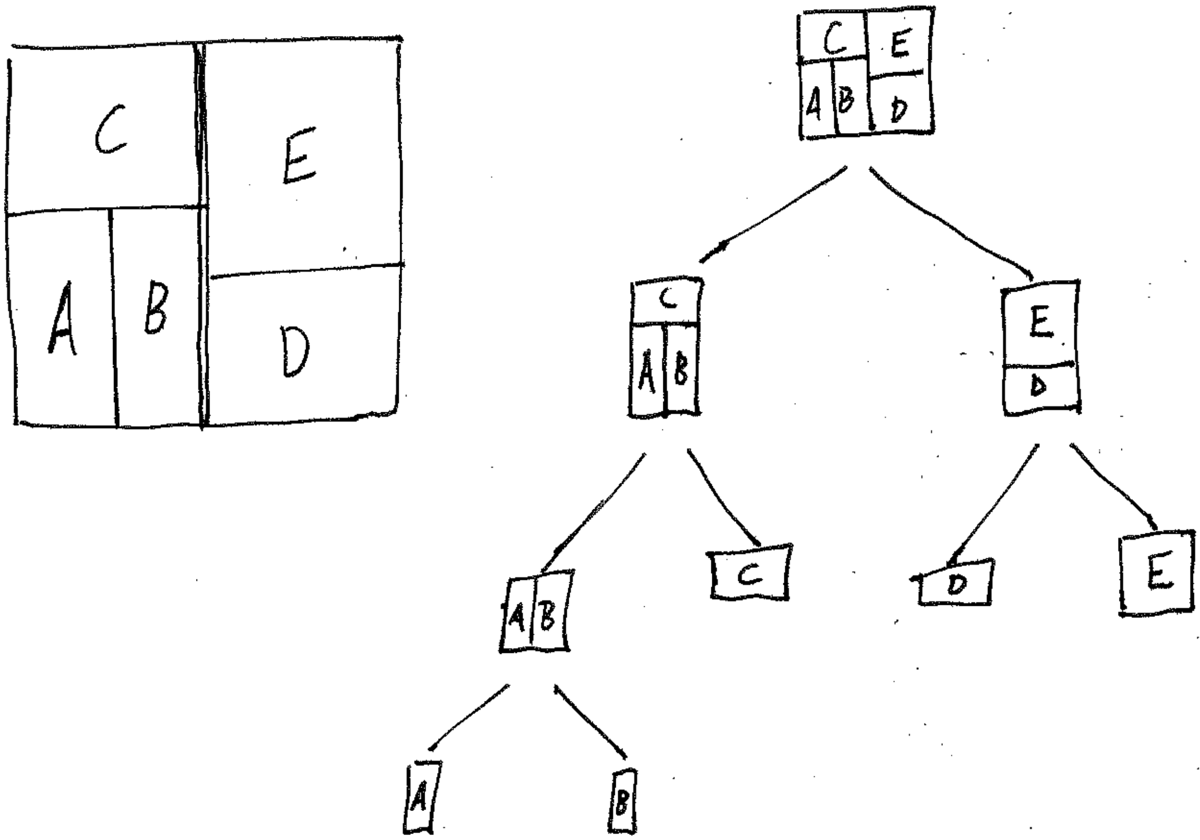


- The nodes of the kd-tree are identified with a region of space.
- The children of a node **N** form a partition of the region N.

A Generic Algorithm for Range Searching

Note: easy to modify this to do range counting. Report returns # leaves.

* return 1
** return 0
*** return G+D

Search (T: tree, R: range)

If T is a single node n
{ If n.point $\in R$, return n.point *
else return $\emptyset$. **
}

If T.left $\subseteq R$, G=Report(T.left)
else if T.left $\cap R \neq \emptyset$, G= Search(T.left, R)
else G= $\emptyset$

If T.right $\subseteq R$, D= Report(T.right)
else if T.right $\cap R \neq \emptyset$, D= Search(T.right, R)
else D= $\emptyset$

return G $\cup$ D ***

Search still works w/o this code

recursive calls

To Implement:

$\subseteq$ Inclusion

$\cap \neq \phi$ Nonempty Intersection

Report

Report (T)

If T is a single node n
Then return n.point

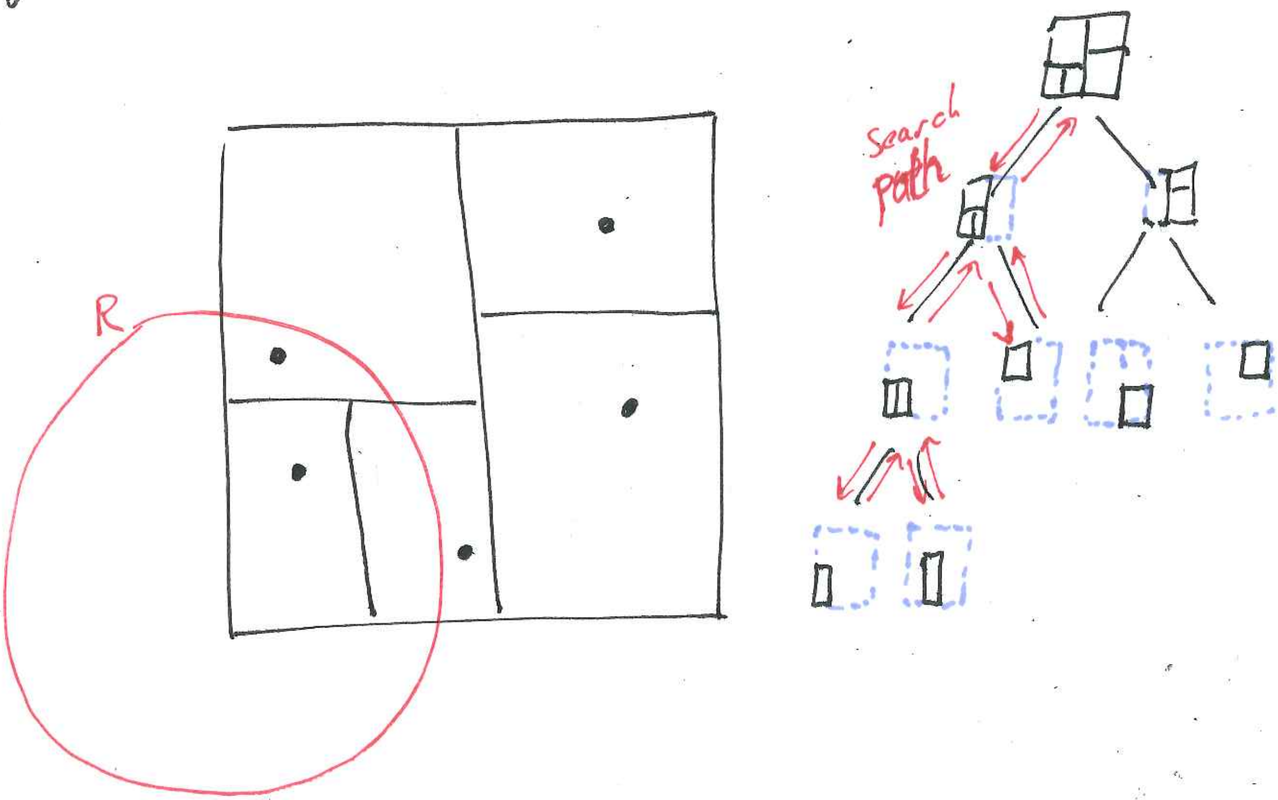
Else
return Report(T.left) $\cup$ Report(T.right)

The algorithm is generic in two ways:

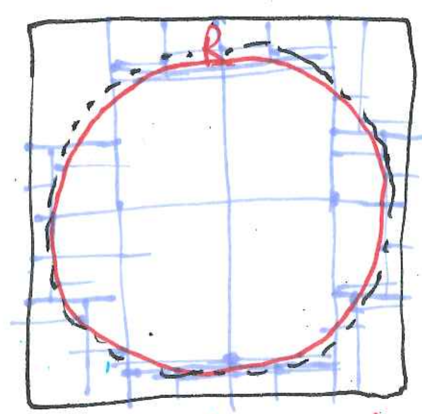
(1) How we partition space.

(2) What kinds of ranges are supported.

Let's look at another popular class of ranges: balls (or disks).



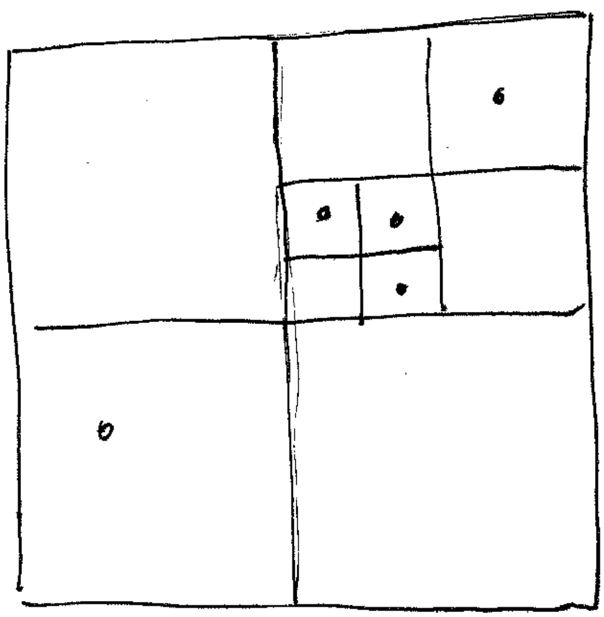
Note: $O(\sqrt{n}+k)$ Analysis for orthogonal ranges does not apply to disks!



R intersects every box but contains no points. Running time $O(n)$.

Another way to partition space: The Quadtree (QT)

Def A Quadtree is a 4-ary tree in which every node is a square and the 4 children of any node decompose it into 4 equal pieces.



Advantages:

- No need to compute medians
- Cells are square (useful for ANN)

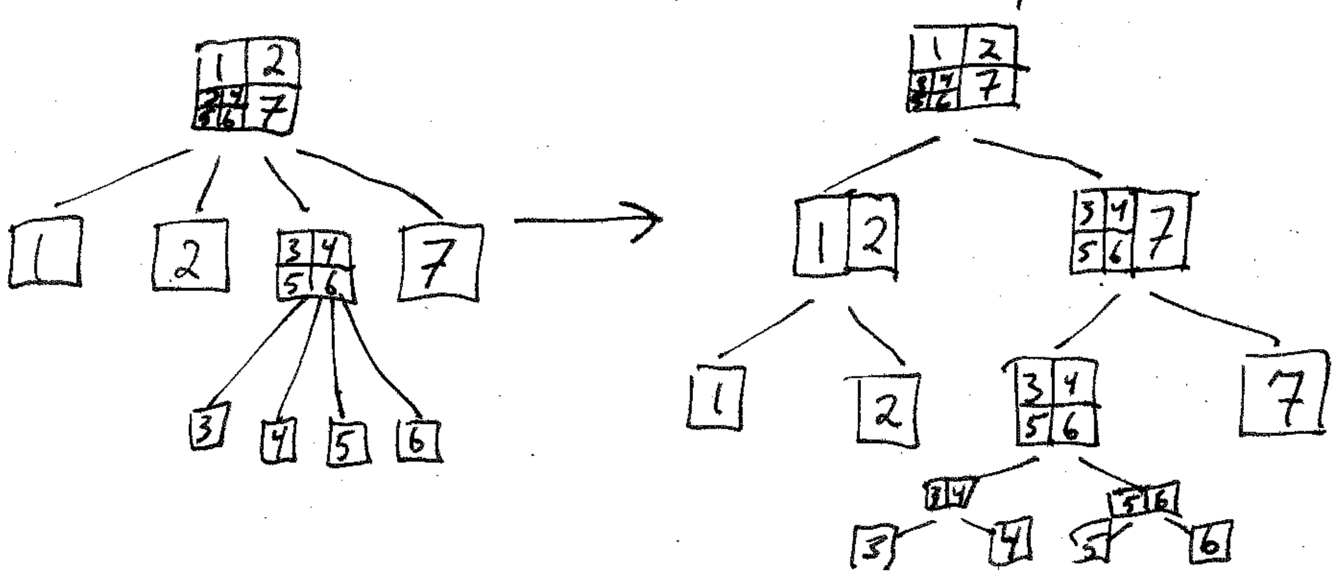
Simple Algorithm:

While any square has more than one point, split it.

~~Smallest node containing all of the~~

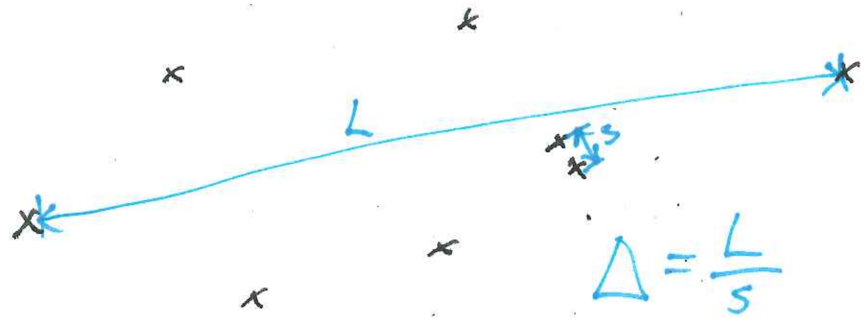
Assume root size ≈ diameter(P)

Note: We can treat the QT like its a binary tree.



<u>Def</u> The <u>spread</u> Δ of a point set P is the ratio of the largest to smallest pairwise distances.

<u>example</u>



$\Delta = \frac{L}{s}$

<u>Claim</u>: The depth of a QT for P is $O(\log \Delta)$.

<u>pf</u> Assuming size (i.e. side length) of root is $O(L)$, observe that at each level the size goes down by a factor of 2.

At level i, size is $\frac{O(L)}{2^i}$

Consider a node whose children have max depth k

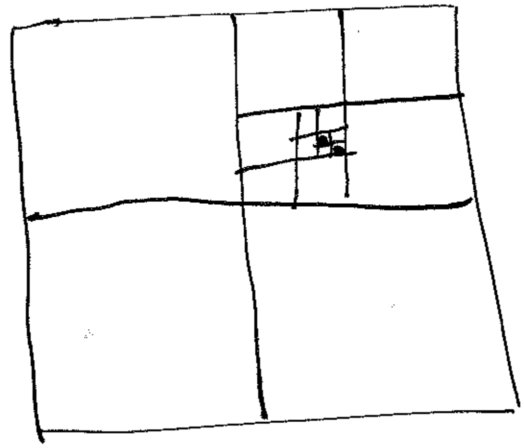
Let p, q be two such children.


$\|p-q\| \leq \frac{\sqrt{2} \cdot O(L)}{2^{k-1}}$

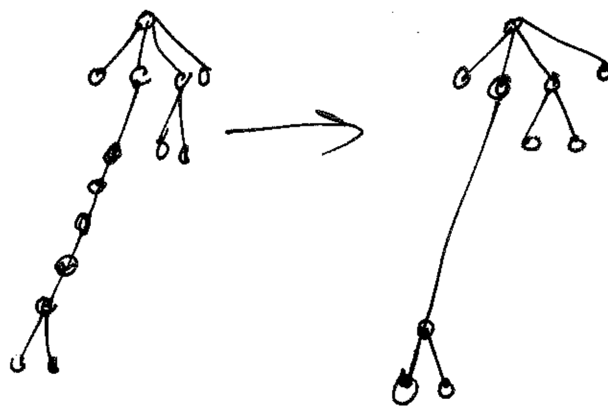
$\Delta \geq \frac{L}{\|p-q\|} \geq \frac{2^{k-1}}{\sqrt{2}\, O(1)} \Rightarrow k = O(\log \Delta)$

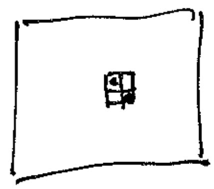
Compressed Quadtrees

The Bad Case: many splits for few points.



<u>The idea</u>: "Compress" paths in the quadtree.





Cells become either a box or a box with a smaller box cut out of it.

<u>Claim</u>: The depth of a compressed quadtree is $O(n)$.