Computational Geometry Homework 2

Administration

Your answers should be typeset in LaTeX or some equivalent and submitted as a **pdf**. The LaTeX sourse of these questions may be found on the course website under "homework". Name your files as "1_*your_last_name*.pdf", all lowercase letters. For example, I would call mine **1_sheehy.pdf**.

Due: Before class, Wednesday, October 22, 2014.

Email Solutions to donald@engr.uconn.edu

1 The HalfEdge Data Structure

For this question, assume that you have a triangulation T of a point set P that is represented by a halfedge data structure. This data structure has a HalfEdge class that supports the following operations on a half edge h.

- $h.v$ is the vertex at the tail of h.
- $h.next$ is the next half edge (going counterclockwise around the face).
- $h.prev$ is the previous half edge (going backwards around the face).
- $h.twin$ is the twin half edge on the other side of the edge.

When creating a new HalfEdge, one passes the vertex of the tail.

1.1 Given a halfedge h, some code below has been started to implement an edge flip. Finish the code by updating all the relevant pointers in the data structure.
flip(h) {
$e = h.prev$
$f = h.next$
$g = h.twin.prev$
$i = h.twin.next$
$j =$ new HalfEdge($g.v$)
$k =$ new HalfEdge($e.v$)
...
...
}

1.2 Using the same style as the previous question, show how to implement the three-way split of the triangle represented by a HalfEdge h by a new point p inside that triangle. That is, the input to the mehtod should be a HalfEdge and a Vertex.

2 Testing the Local Delaunay Condition

2.1 Suppose that you have a triangulation T of a point set P that is represented by a halfedge data structure as in the previous problem. Let $e = \overline{ab}$ be an edge of T and let c and d be vertices of P such that the triangles containing e are $\triangle abc$ and $\triangle abd$. Recall e is locally Delaunay (LD) if it is an edge of the Delaunay triangulation T' of the four points $\{a, b, c, d\}$. Prove that it only requires *one* InCircle test to check if e is LD.

2.2 Show how to implement the LD test using the HalfEdge data structure. That is, for a given halfedge, return true iff it is LD.

3 More on Delaunay Triangulation

3.1 Given a set of points P in general position, the Gabriel graph of P is the set of edges $\overline{ab}$ such that the circle with diameter $\overline{ab}$ contains no point of P in its interior. Prove that the Gabriel Graph is always a subset of the Delaunay triangulation.

3.2 The Euclidean Minimum Spanning Tree T of a point set P is the *connected* graph whose vertices are the points of P and such that

$$\sum_{\overline{ab} \in T} \|a - b\|$$

is minimized. Prove that the Euclidean Minimum Spanning Tree of P is a subset of the Delaunay triangulation of P. Again, assume P is in general position. Hint: Consider a well-known greedy algorithm for mininum spanning trees.