Computational Geometry Homework 3

Administration

Your answers should be typeset in LaTeX or some equivalent and submitted as a **pdf**. The LaTeX sourse of these questions may be found on the course website under "homework". Name your files as "3_*your_last_name*.pdf", all lowercase letters. For example, I would call mine **1_sheehy.pdf**.

    Due by email: Before 10:00am, Friday, November 21, 2014.
    Email Solutions to donald@engr.uconn.edu

1   Tutte's Algorithm

Suppose we wanted to model Tutte's Graph Drawing algorithm as a set of springs that are all pulling on one another, but the spring constants are not all the same. That is, for each edge $\{u, v\} \in E$, let $k_{\overline{uv}} > 0$ be the spring constant for that edge. So, the force acting on any vertex v in a drawing is

$$F(v) = \sum_{u \sim v} k_{\overline{uv}}(u - v).$$

Note that the condition $k_{\overline{uv}} > 0$ is very important in all of the following problems.

1.1 This version of the problem can also be solved using linear algebra. Write down an $n \times n$ matrix L such that $LP = F$ where P is the $n \times 2$ matrix of coordinates for the points and F is the $n \times 2$ matrix of forces. Express your answer by giving a formula for L_{ii} and a formula for L_{ij} for $i \neq j$.

1.2 Show that in any such embedding, the equilibium condition $F(v) = 0$ for interior vertices v implies that

$$v = \frac{\sum_{u \sim v} k_{\overline{uv}} u}{\sum_{u \sim v} k_{\overline{uv}}}.$$

1.3 Show that in such an embedding, every interior vertex is in the convex closure of its neighbors.

1.4 Show that in such an embedding, there are monotone paths from any interior vertex p to the boundary in every direction v. That is, show that for all nonzero vectors $v \in \mathbb{R}^2$ and all interior vertices p, there is a neighbor q of p such that $v^\top(q - p) \geq 0$. Note that your proof should use linear algebra and a previous problem.

2   Project Checkpoint

2.1 Create a git repository for your group. Make it either private or public. Share it with me. There is an option to do this on `github.uconn.edu`. My username is `she13001`.

2.2a (Only if you are doing the project)
Give a short description of how your project will look to a user. Including a screen shot of work in progress

would be great. Be sure to explain what the user will see and more importantly, what they will learn by using your project. Add a readme file (preferably in markdown) to appear on the github page.

2.2b (Only if you are doing the scribe notes)
Commit an outline of the notes to the git repository.