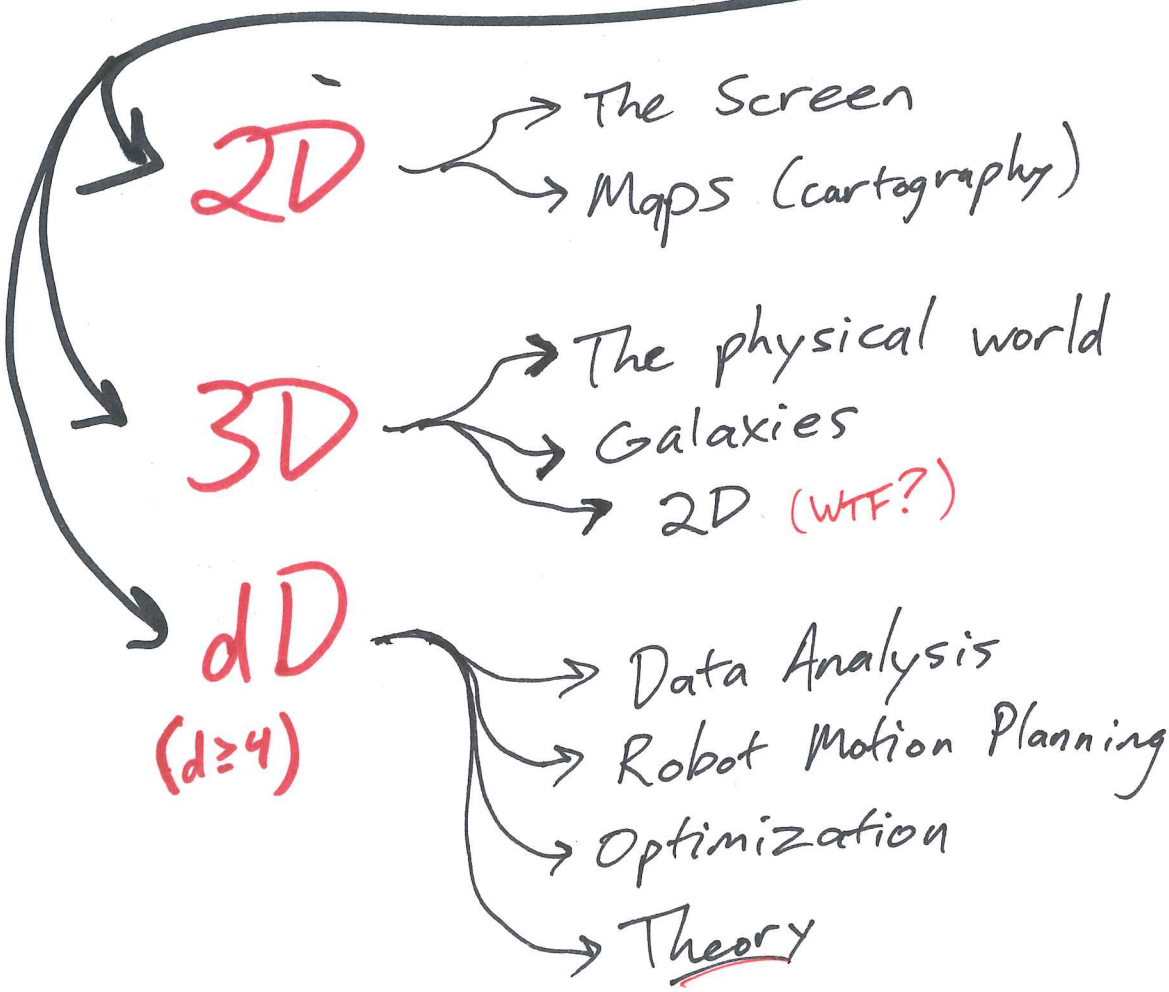
What is Computational Geometry?

- Algorithms & Data Structures for
  - storing
  - organizing
  - searching

  } geometric objects



GEOMETRIC { Input / Output / Queries

FOUNDATIONS: discrete/combinatorial geom.
linear algebra
topology

you will learn to love this stuff

Why study Computational Geometry?

HIGH LEVEL SKILLS:

#1 Visual Intuition, Physical Intuition → Rigor

#2 Abstract Abstract Thinking (not a typo)
- separate geometry, topology, + combinatorics

#3 Understand Algorithms beyond 1D.

Why Comp. geom. cont.

SPECIFIC PROBLEM DOMAINS:

#1 Foundational CG Questions

- Convex Hulls
- Voronoi Diagrams
- Delaunay Δ^ns

Stuff every computer scientist should know.

"triangulations"

#2 Planar Graphs

- Interplay of geom/top/comb.

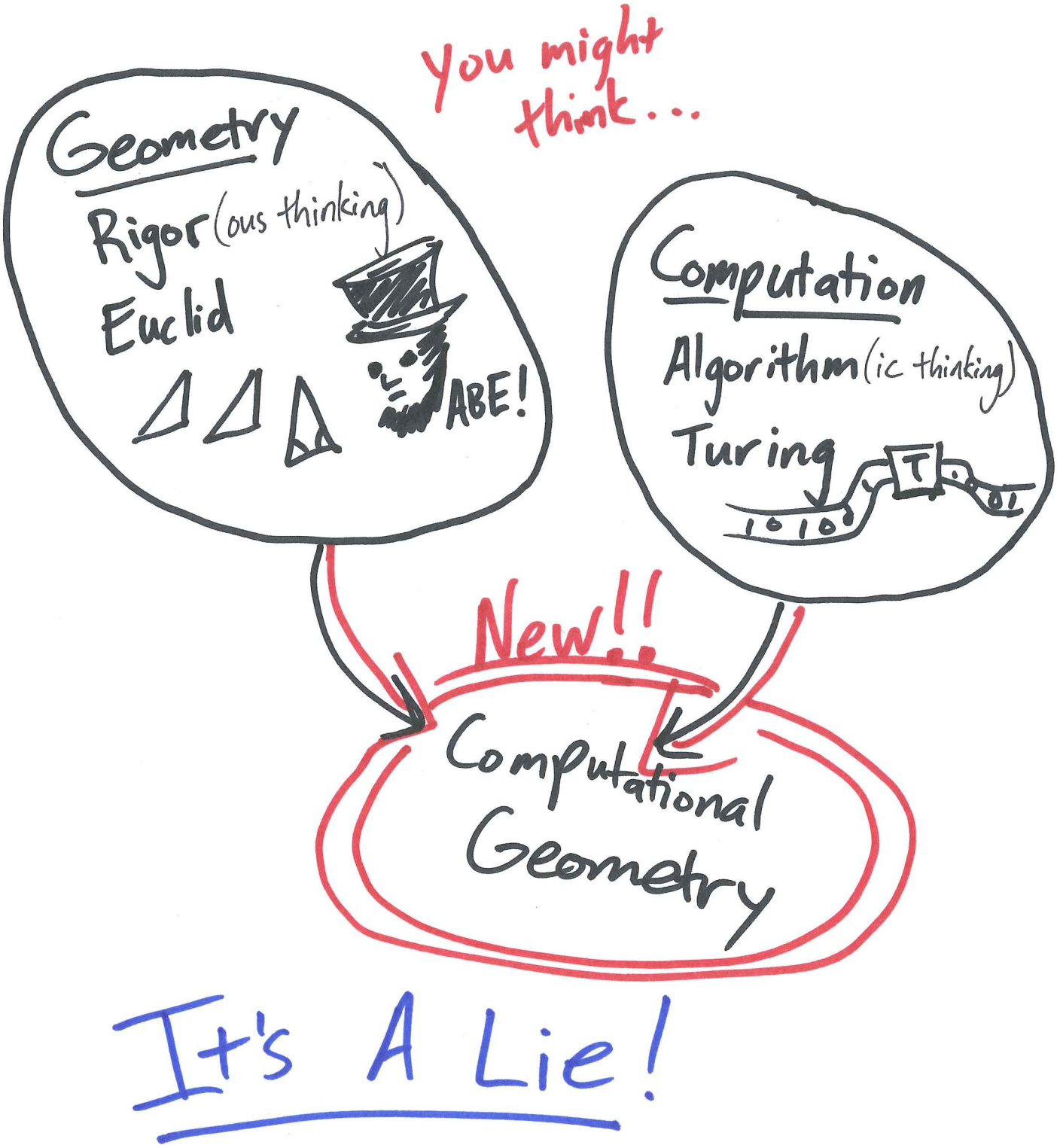
#3 Search

- Nearest Neighbor Search
- Range Searching
- Approximations

#4 Optimization

- Linear Programming
- Core Sets

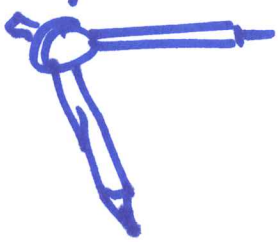
The geometric view


You might think...
Geometry
Rigor(ous thinking)
Euclid
ABE!
Computation
Algorithm(ic thinking)
Turing
T
1 0 1 0
New!!
Computational Geometry
It's A Lie!

Geometry has always been Computational

Euclid's Model of Computation:

Ruler and Compass



First 3 Axioms:

Ruler
- (1) Draw a line thru any 2 points
- (2) Extend any line

Compass
- (3) Draw a circle with fixed center and radius.

QED

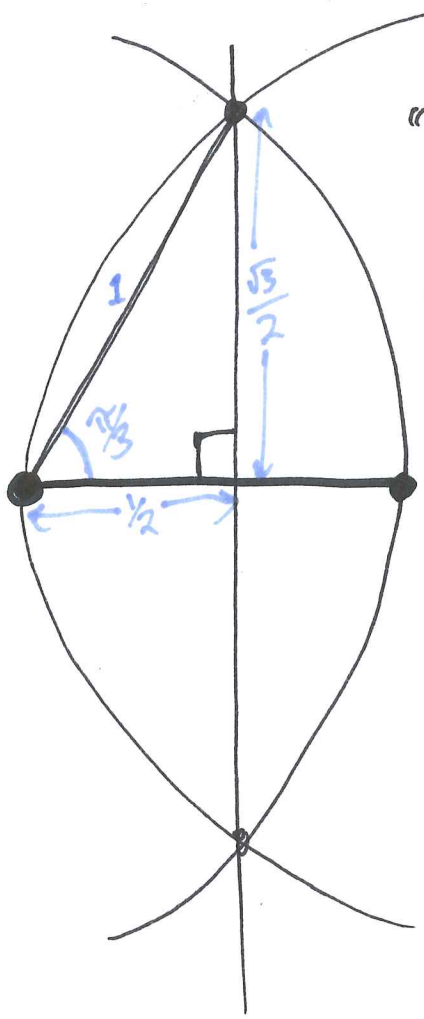
↑

"It has been demonstrated"

QEF

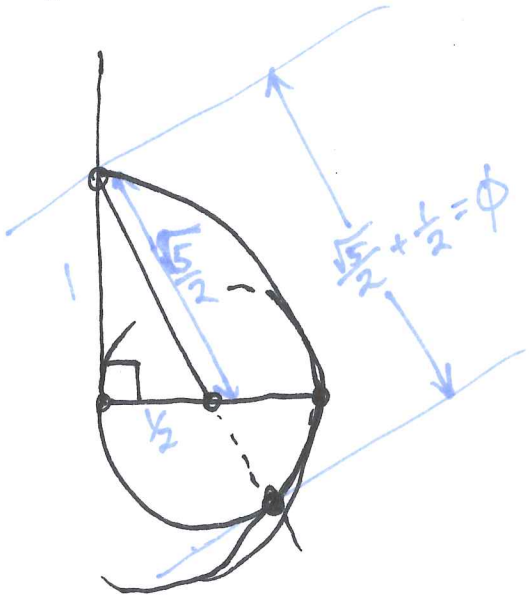
↑

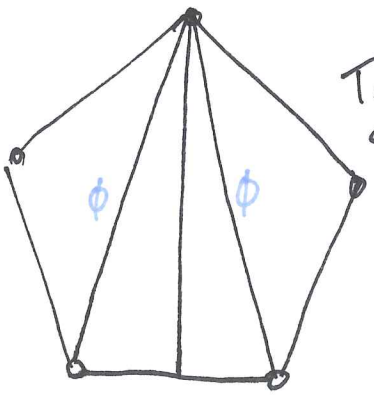
"It has been constructed"



"Compute" lengths + angles

"construct" the regular triangle



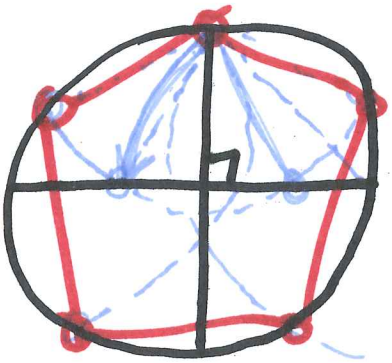


The Regular Pentagon

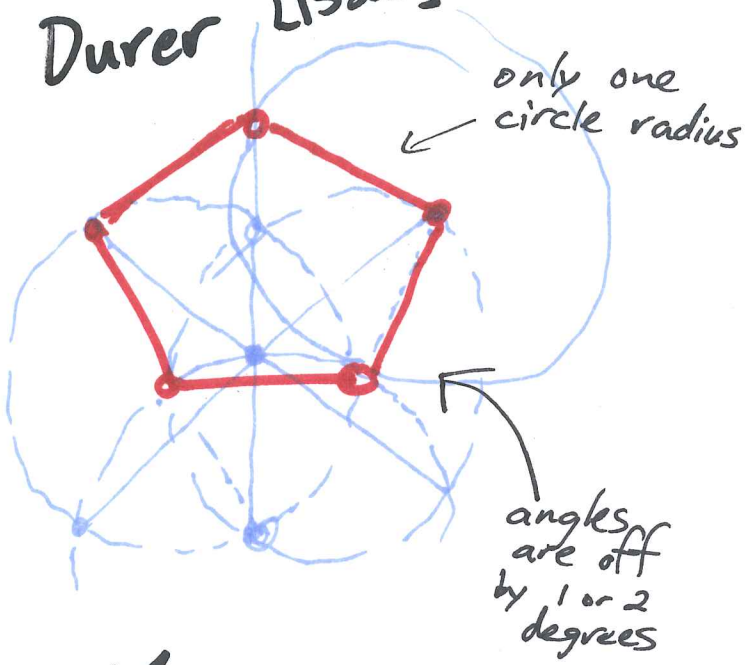
~~Hexagon~~

Two more Pentagon Constructions

Ptolemy ~~Euclid~~ [4th century after Euclid]



Durer [1525]



This illustrates 3 important themes

#1 PRECISION

#2 APPROXIMATION

#3 DRAWINGS LIE

Where is the Big-O?

Usually: input size n.

~~output~~ running time $O(f(n))$.

Perhaps ...

Given n, a prime number,
Construct a regular n-gon

How many steps?

How about a 7-gon?

oops, it's impossible.

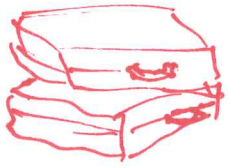
Theme #4 Geometric Structure Matters

What regular polygons can we construct?

Gauss 1767: If all the odd prime factors of n are Fermat primes ($2^{2^k}+1$) then there exists a R & C construction of the n-gon.

Wantzal 1837: There exists a R&C construction of the n-gon iff all odd prime factors of n are Fermat primes.

k	$2^{2^k}+1$	
0	3	
1	5	
2	17	← Gauss
3	257	← Friedrich Julius Richelot [1832]
4	65,537	← Johann Gustav Hermes [1896]



Summary

Geometric Algorithms & D.S.

Classic Model of Computation

Ruler + Compass

Themes

#1 Precision

#2 Approximations

#3 Drawings Lie

#4 Geometric Structure Matters

Course ~~Overview~~ Administrivia

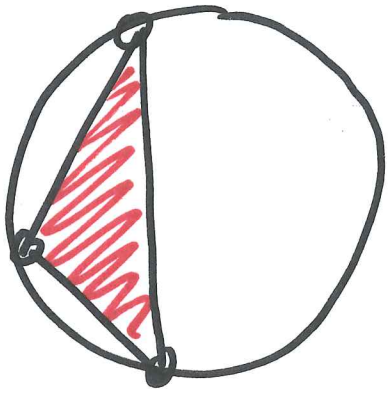
website donsheehy.net/....

studentadmin... probably

grades (tentative)

- 10% Class Participation
- 30% HW
- 10% Midterm
- 20% Final
- 30% Project ← If it's not fun you're doing it wrong

For Next Time



Given 

Find 

Find a ruler + compass construction for the circumcircle of a triangle.

Bonus: Draw a triangle for which the R&C construction of the circumcircle is "tricky."