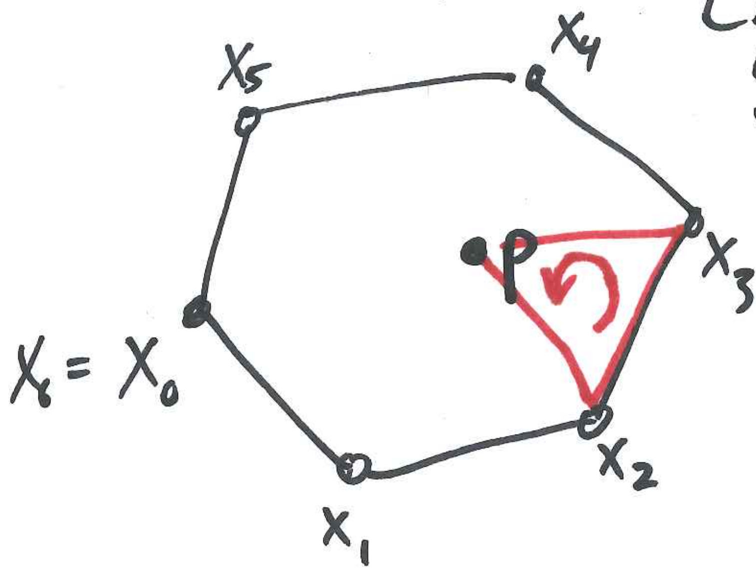
(Convex) Polygons

Test if a point p is "inside" a convex polygon:

Check if $\exists\, x_i x_{i+1} p = 1$ for each edge $x_i x_{i+1}$



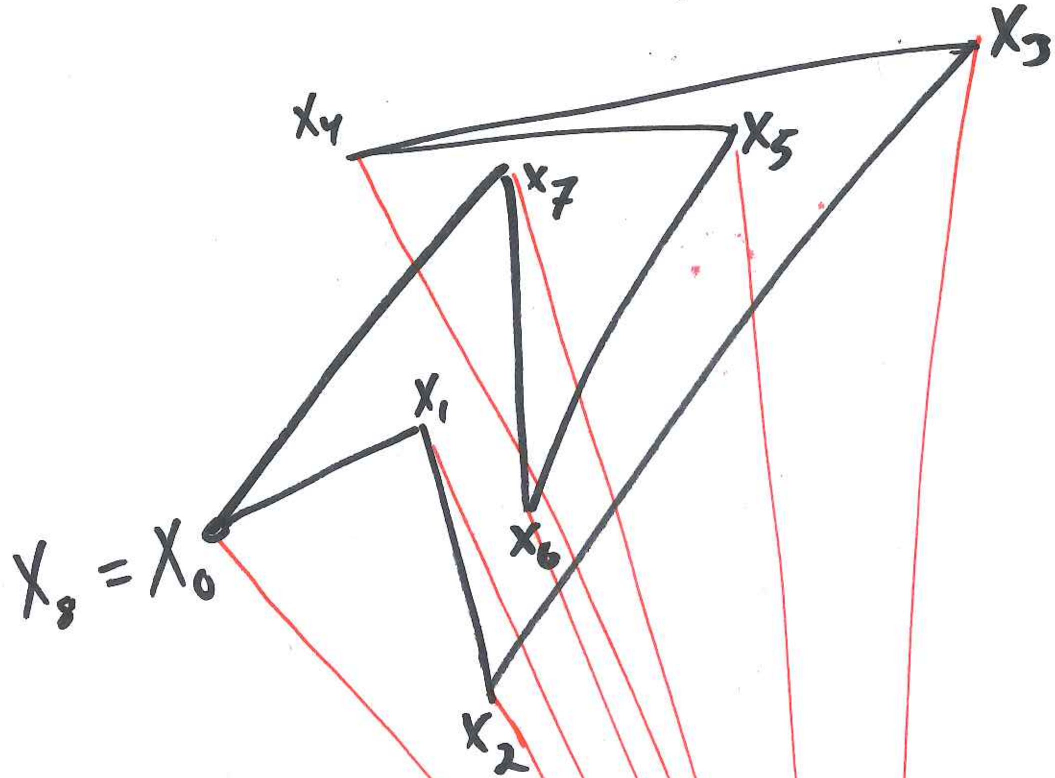
We can also compute area this way:

$$\text{Area} = \frac{1}{2}\sum_{i=0}^{n-1} \det\left[x_i - p \quad x_{i+1} - p\right]$$

Why $\frac{1}{2}$?

assumes p is inside?

Simple Polygons A polygon is simple if no edges intersect (except consecutive edges at their common endpoint.)



$$\text{Area} = \left| \frac{1}{2} \sum_{i=}^{n-1} \det \begin{bmatrix} x_i - p & x_{i+1} - p \end{bmatrix} \right|$$

Why? Inclusion-Exclusion Principle

Jordan Curve Thm for Polygons

Given a simple polygon P, $\mathbb{R}^2 \setminus P$ has 2 connected pieces (components) one is bounded, the other is unbounded, and P is the boundary of both.

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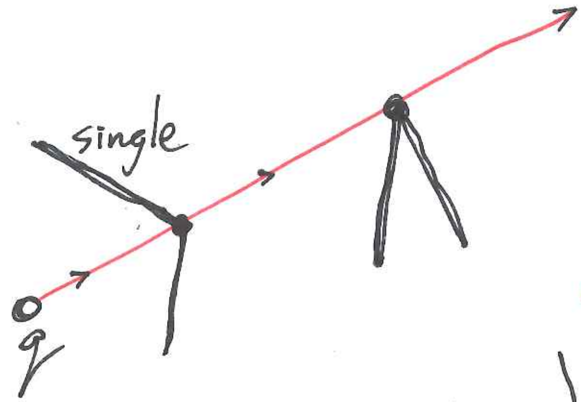
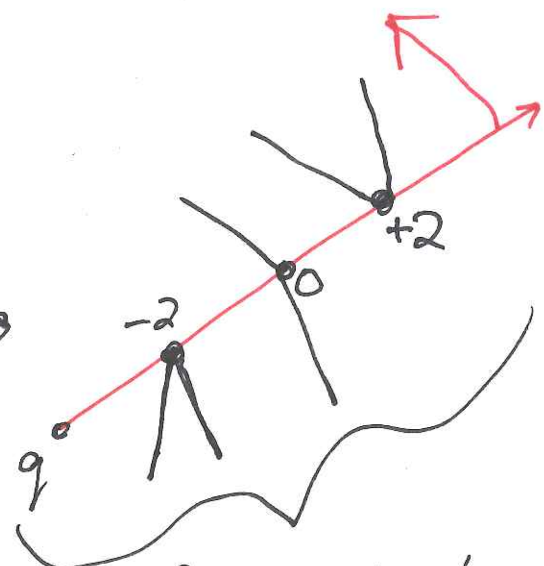
pf

(1) For any $q \in \mathbb{R}^2 \setminus P$, count the intersections between P and a ray emanating from q.



(2) Rotate the ray around q. The count only changes when we cross a vertex. Three cases



So count stays (3) odd or even. Cannot change parity.

$\Downarrow$

(4) define parity(q)

(3) Observe that if q, q' are connected by a straight line that doesn't touch P, then parity(q) = parity(q').