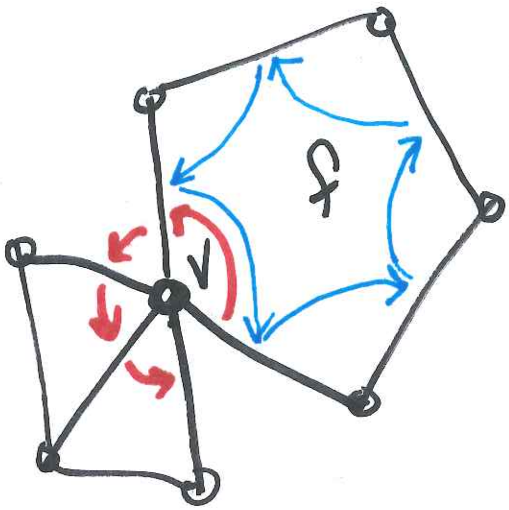
PSLG
data structures

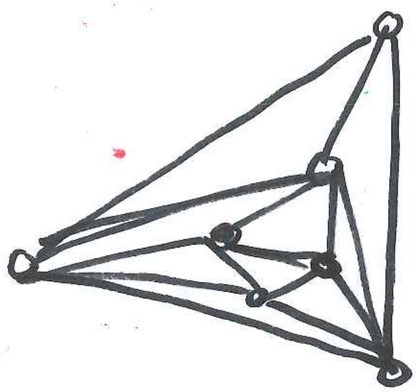


Incidence operations
- edges incident to f.
- edges incident to v.

symmetric

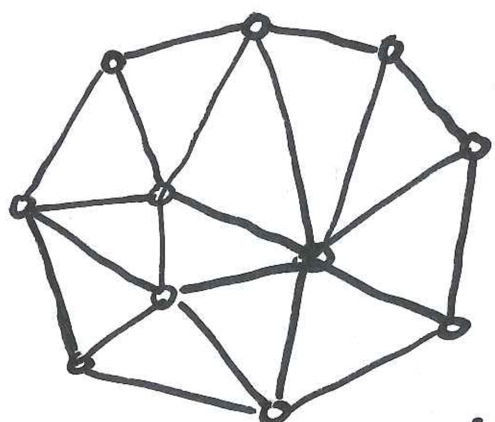
- 1-connected ⇒ no holes
- 2-connected ⇒ polygonal faces
- 3-connected ⇒ Uniquely defined faces (independent of drawing)

Def A ***Triangulation*** is a PSLG in which every face is a triangle.



n vertices $\Rightarrow$ $2n-4$ faces
$3n-6$ edges

(Sometimes we allow the outer face to be non-triangular.)



h vertices on the hull
k vertices on the interior $\Bigg\} \Rightarrow 2k+h-2$ triangles

By Euler's Formula
$(E = \frac{3}{2}F)$

Note: Triangulations are 3-connected. (Prove it!)

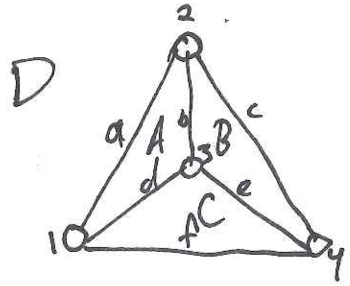
Posets and Duality

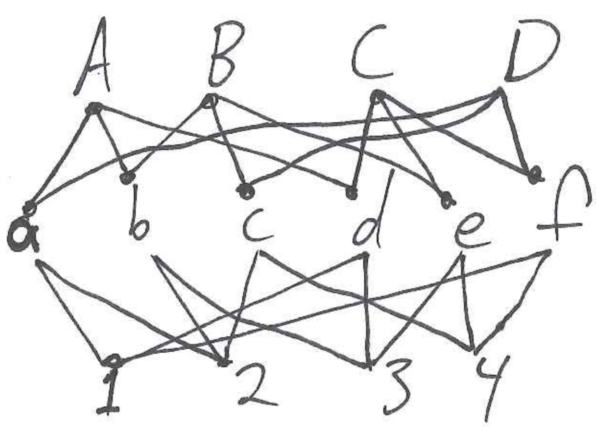
Incidence puts a partial order on edges, vertices, + faces.

$$\underset{\text{vertex}}{v} \prec \underset{\text{edge}}{e} \quad \text{iff} \quad v \in e$$

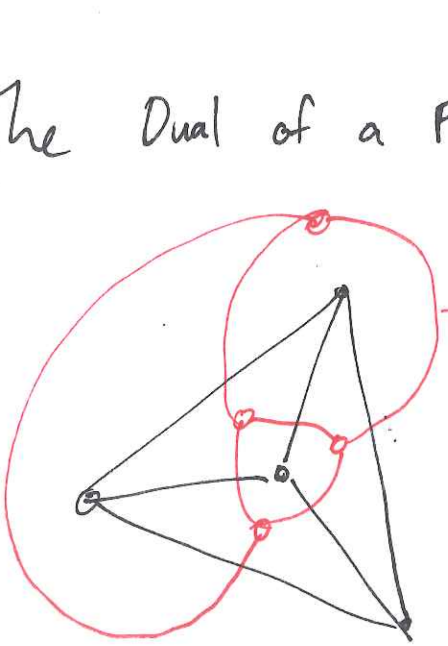
$$\underset{\text{edge}}{e} \prec \underset{\text{face}}{f} \quad \text{iff} \quad e \in bdy(f)$$

The Poset (partially ordered set) or Hasse Diagram illustrates all the incidence relations.





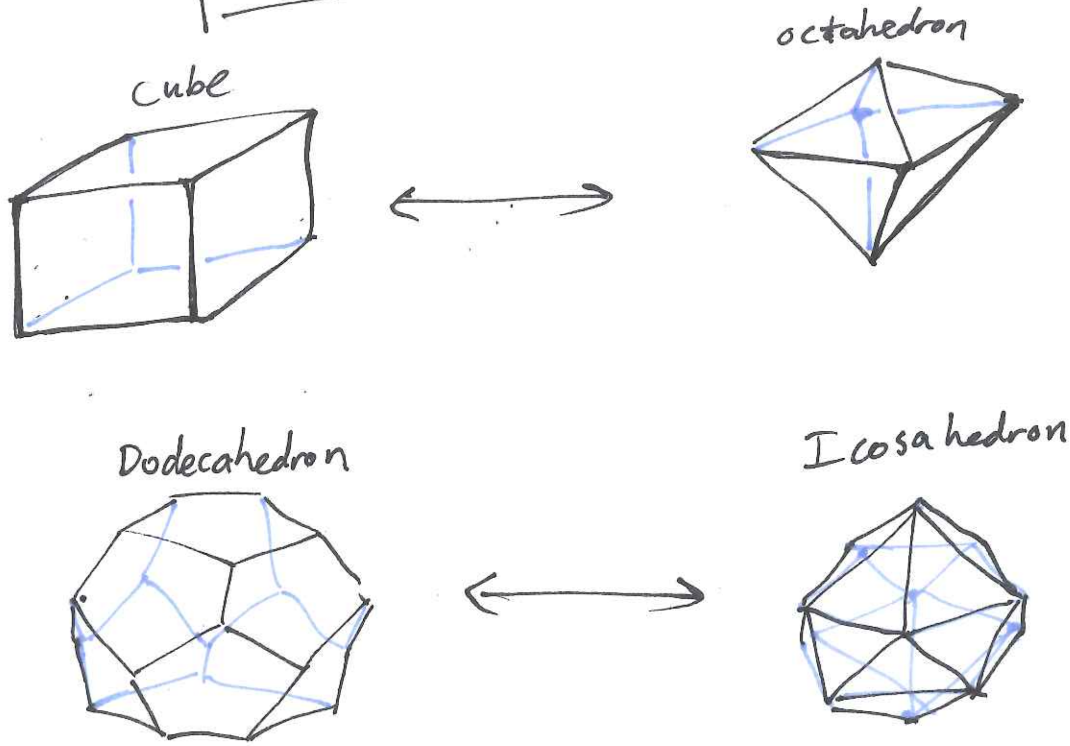
The Dual of a PSLG has $\begin{cases} \text{a vertex for each face} \\ \text{a face for each vertex} \\ \text{an edge for each edge} \end{cases}$



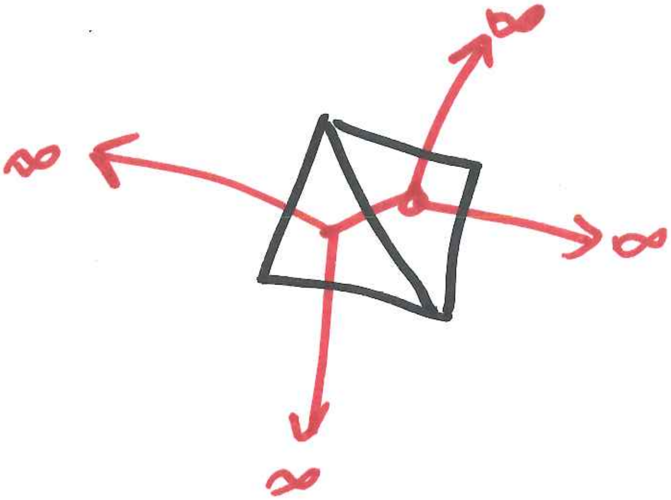
$\boxed{\text{If the graph is 2-connected,}}$ we get the dual by turning the POSET upside-down.

Why is it useful to look only at this case?

Famous Duals



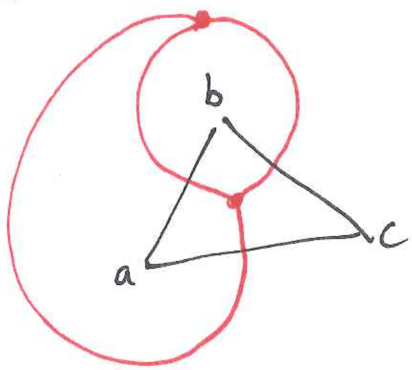
A Convenience: Put the dual vertex to the outer face "at infinity."

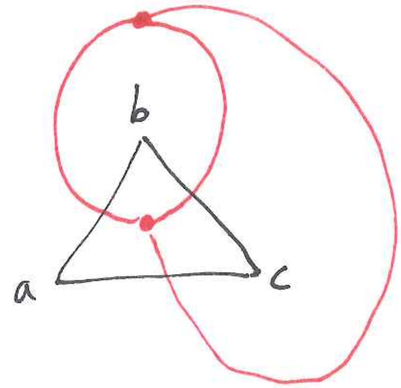


So, the underlying space is not $\mathbb{R}^2$, but rather $\mathbb{R}^2 \cup \{\infty\}$

It's the sphere S_2

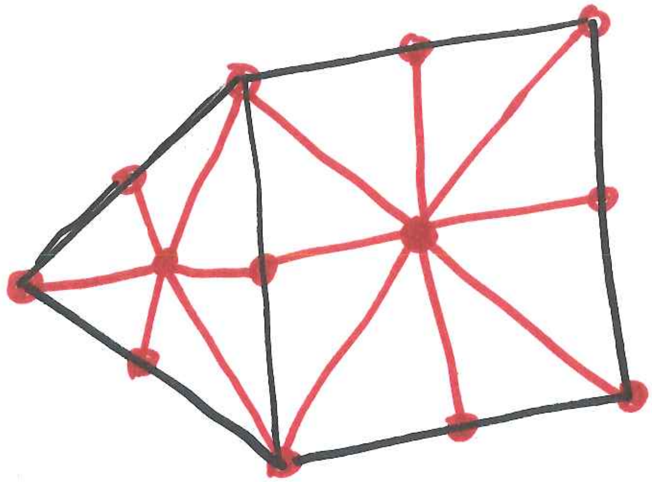
Which face is the infinite face in the dual?





It could be any vertex of the CH.

Barycentric Decomposition



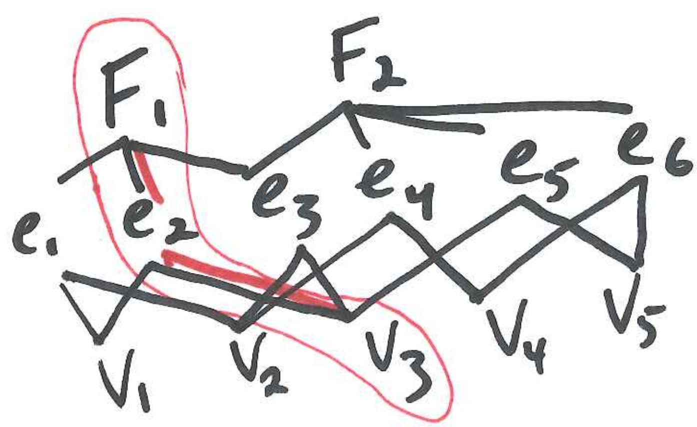
A vertex for each cell (V, E, or F), a triangle for each $(v,e,f) \in V \times E \times F$ s.t. $v < e < f$.

For now, we assume 2-connectivity so that the barycentric decomposition is a <u>triangulation</u> (i.e. all faces are triangles).

ex) Not 2-connected

outer face is a quad

In the Poset, A barycentric triangle is a maximal chain.

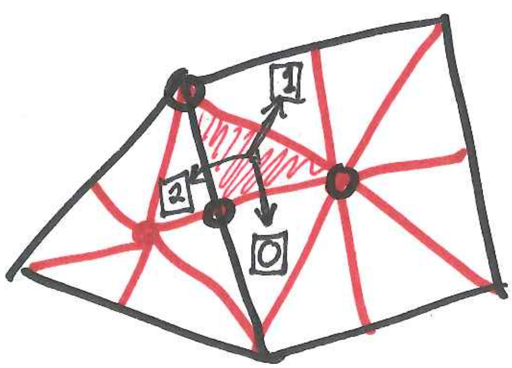


The barycentric decomposition encodes both primal and dual.

(because it encodes the poset)

$$\begin{tikzcd} F \arrow[d, dash] & & V^\circ \arrow[d, dash] \\ e \arrow[d, dash] \arrow[rr, squiggly, "\text{dual}"] & & e^\circ \arrow[d, dash] \\ V & & F^\circ \end{tikzcd}$$

We can navigate the BD

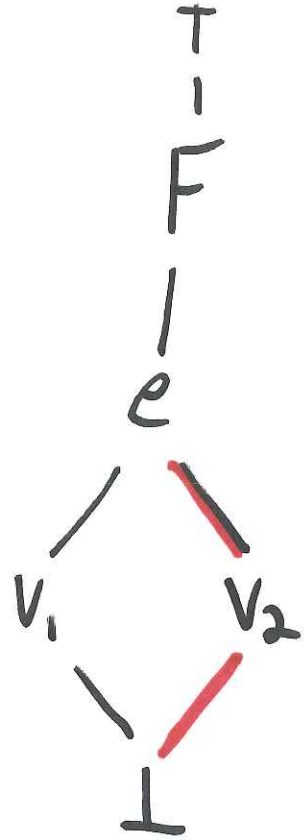


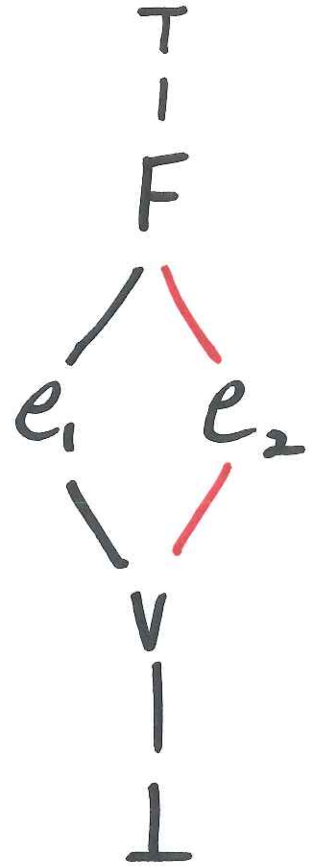
handles are triangles

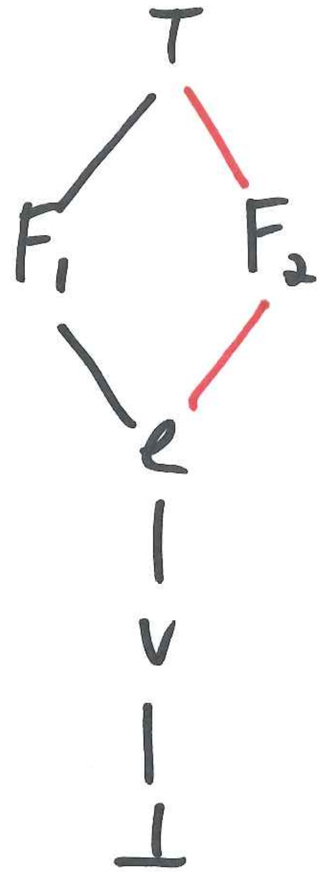
3 moves

- change vertex $\boxed{0}$
- change edge $\boxed{1}$
- change face $\boxed{2}$

How do these operations relate to the $\underbrace{\text{doubly connected edge}}_{\text{a.k.a. half-edge data structure}}$ list?







0 1 2

Diamonds in the Hasse Dgm indicate adjacency in the BD.

Polyhedral Complex

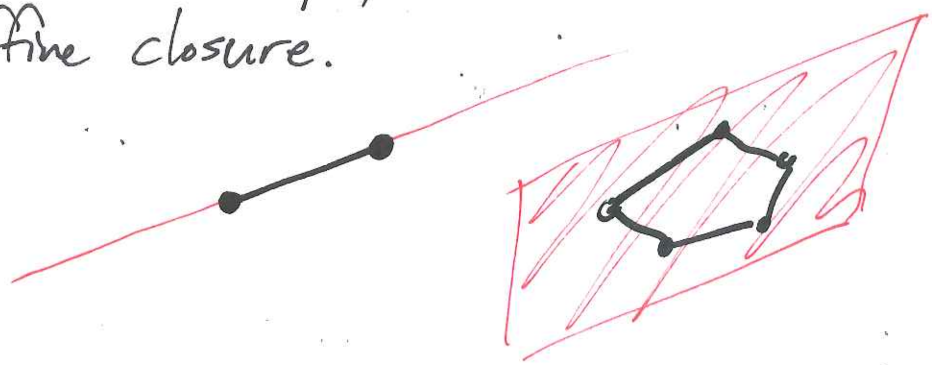
<u>Def</u> A <u>polyhedron</u> is the intersection of a finite set of closed halfspaces.

Caveat: Could be unbounded.

ex) In $\mathbb{R}^2$ it's a polygon (or a line segment).

ex) Cubes in $\mathbb{R}^3$

<u>Def</u> the <u>dimension</u> of a polyhedron is the dimension of its affine closure.



Some facts

~~Def~~ The boundary of a polyhedron is the union of polyhedra of one dimension lower.

So a polyhedron also has a poset.

<u>Def</u> A polyhedron contained in the boundary of polyhedron P is called a <u>face</u> of the polyhedron P.

A polyhedral complex is a set of polyhedra that is closed under taking boundaries and for any pair of polyhedra P_1, P_2

$$P_1 \cap P_2 = \left\{ \emptyset \quad \text{or} \quad \begin{array}{l}\text{a common face} \\ \text{of } P_1, P_2.\end{array} \right\}$$