Polyhedral Complex

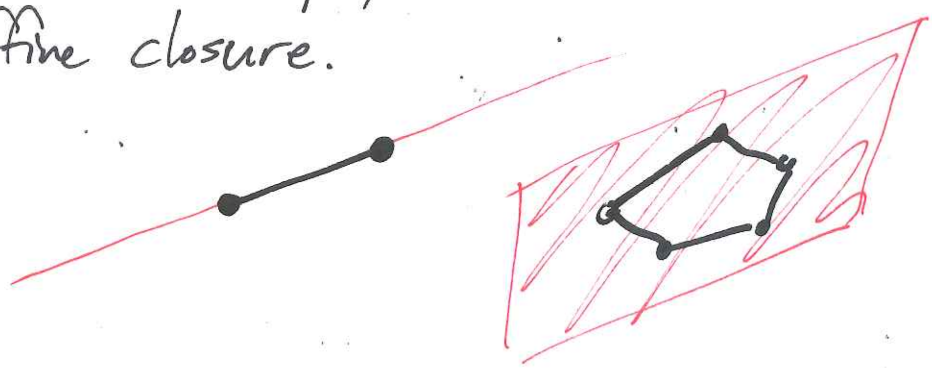
Def A polyhedron is the intersection of a finite set of closed halfspaces.

Caveat: Could be unbounded.

ex) In $\mathbb{R}^2$ it's a polygon (or a line segment).

ex) Cubes in $\mathbb{R}^3$

Def the dimension of a polyhedron is the dimension of its affine closure.



Some facts

~~Def~~ The boundary of a polyhedron is the union of polyhedra of one dimension lower.

So a polyhedron also has a poset.

Def A polyhedron contained in the boundary of polyhedron P is called a face of the polyhedron P.

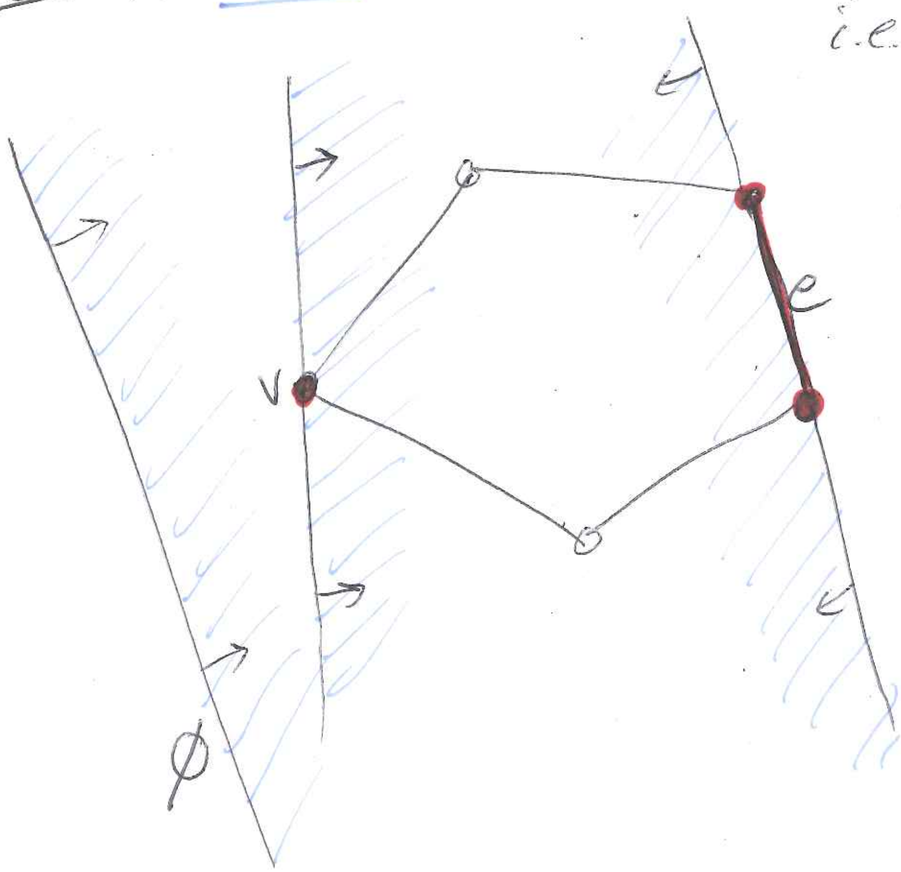
A polyhedral complex is a set of polyhedra that is closed under taking boundaries and for any pair of polyhedra P_1, P_2

$$P_1 \cap P_2 = \left\{ \emptyset \quad \text{or} \quad \begin{array}{l} \text{a common face} \\ \text{of } P_1, P_2. \end{array} \right\}$$

Def For a convex set S, a <u>supporting hyperplane</u> H is one that has all of S in one closed halfspace bounded by H.

Def A <u>face</u> of a polyhedron P is the intersection of P with a supporting hyperplane. We also say (by convention) that P is a face of itself.

Def A <u>facet</u> is a face of <u>codimension 1</u>.
i.e. dimension $= d-1$.



Note: The polyhedral complex gives a poset

Def For a polyhedral complex K, the underlying space $|K| = \bigcup_{P \in K} P$.

Def We'll say K is a decomposition of S if $S = |K|$.

Convex Decomposition from a point set

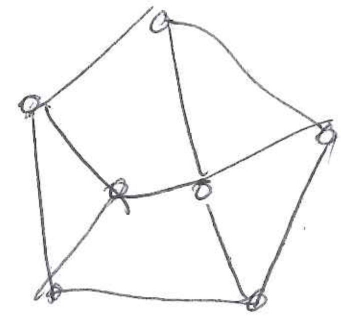
Input: $P \subset \mathbb{R}^2$

Output: Polyhedral Complex K s.t.

$P =$ 0-faces of K and

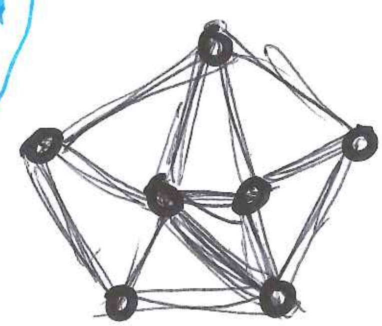
$|K| = CC(P)$.

convex closure

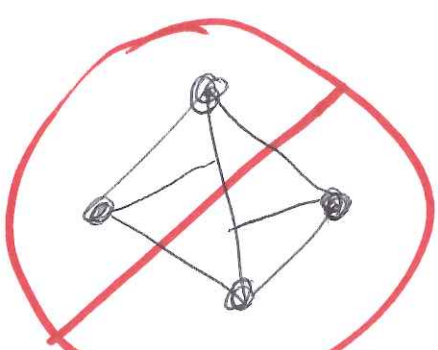


We say a convex decomposition of P is a triangulation of P if every 2-face is a triangle.

Sometimes denoted Δ^n



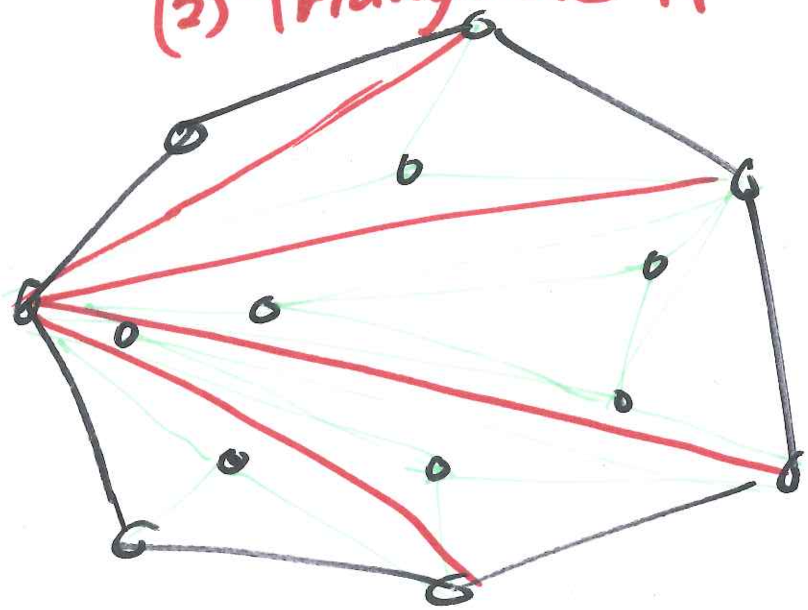
Note: for $P \subset \mathbb{R}^d$, we say K is a Δ^n if every d-face is a $\underbrace{\text{simplex}}_{\text{convex closure of } d+1 \text{ affinely independent points}}$.



Triangulations of point sets exist.

Idea: (1) Start w/ CH(P)

(2) Triangulate it



(3) Repeatedly split Δs 3 ways at new vertices