Today's Lecture: More on projective duality.

Last Time:

In $\mathbb{R}^2$: point $\begin{bmatrix} p_x \\ p_y \end{bmatrix} \xleftrightarrow{\text{duality}}$ line $y = \underbrace{2p_x}_{\text{slope}=2p_x} x \underbrace{- p_y}_{\text{y-intercept}=-p_y}$

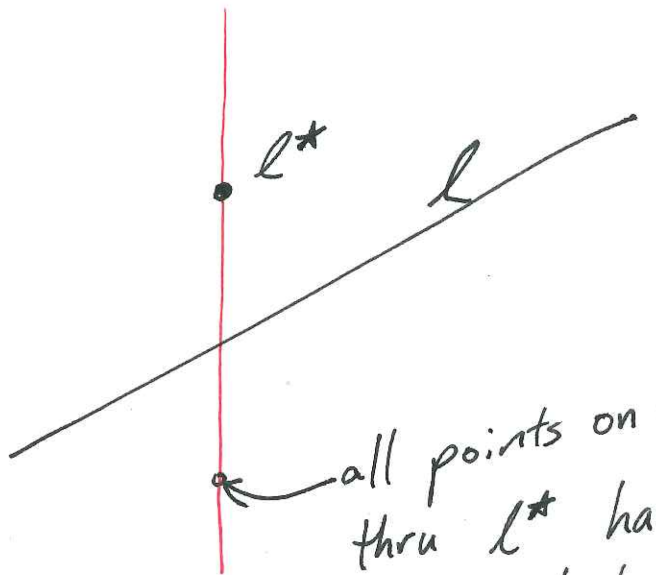
In $\mathbb{R}^3$: point $\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \xleftrightarrow{\text{duality}}$ plane $z = 2p_x x + 2p_y y - p_z$

To do higher dimensions, continue the pattern:

point $\begin{bmatrix} p \\ p_d \end{bmatrix} \longleftrightarrow$ hyperplane $x_d = 2p^T x - p_d$

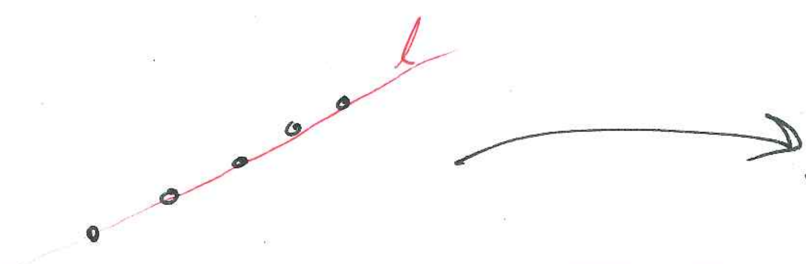
$p \in \mathbb{R}^{d-1}$ $\quad p_d \in \mathbb{R}$ $\quad x_d \in \mathbb{R}$ $\quad x \in \mathbb{R}^{d-1}$

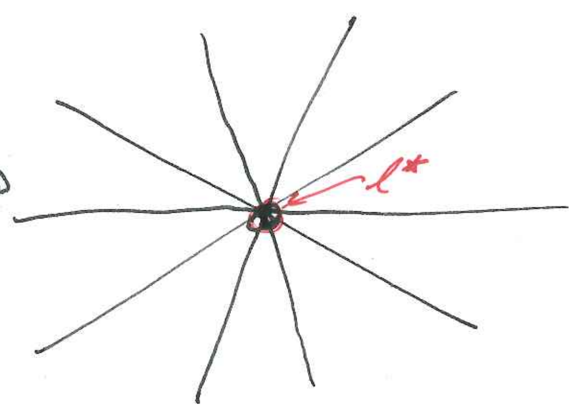
Exercise 1 Given a (nonvertical) line ℓ in the plane. Describe the dual of the set of all lines parallel to ℓ



all points on the vertical line thru ℓ^* have the same slope in the dual as ℓ and thus are parallel to ℓ.

Exercise 2 Given a collection of collinear points, what can be said about the dual collection of lines.



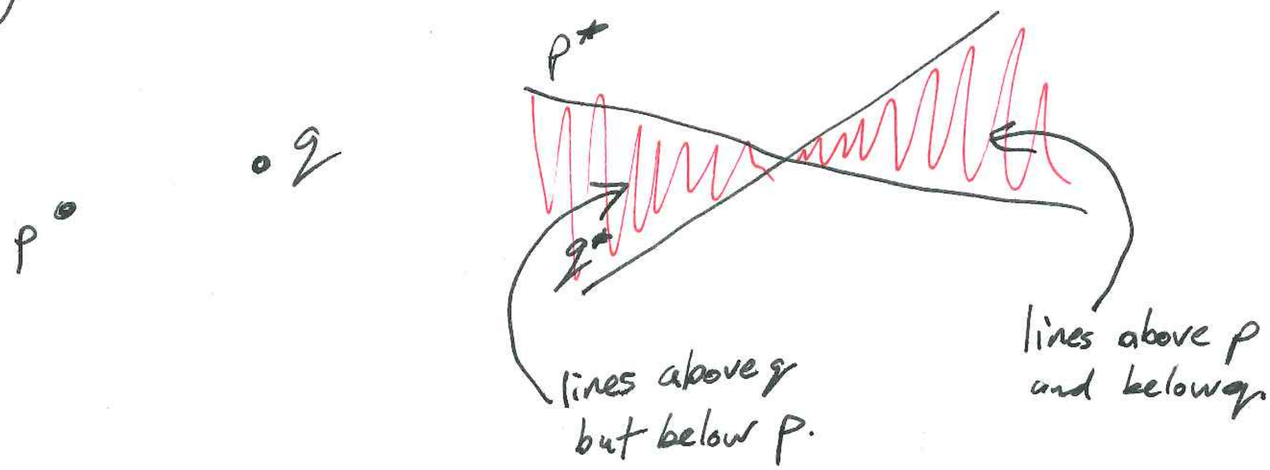


Exercise 1+2 ~~implies~~ together imply that the dual to a vertical line should be a point at infinity.

The dual lines all pass thru ℓ^*.

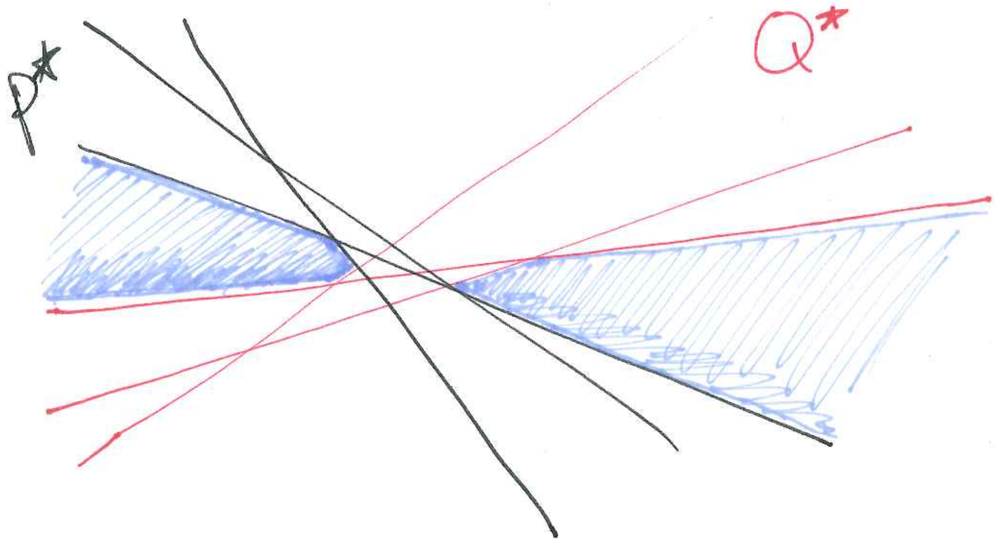
Exercise 3

Given two points p, q, use duality to describe the set of lines passing between them.



Exercise 4

Given two sets of points P and Q, use duality to describe the set of lines separating P and Q.

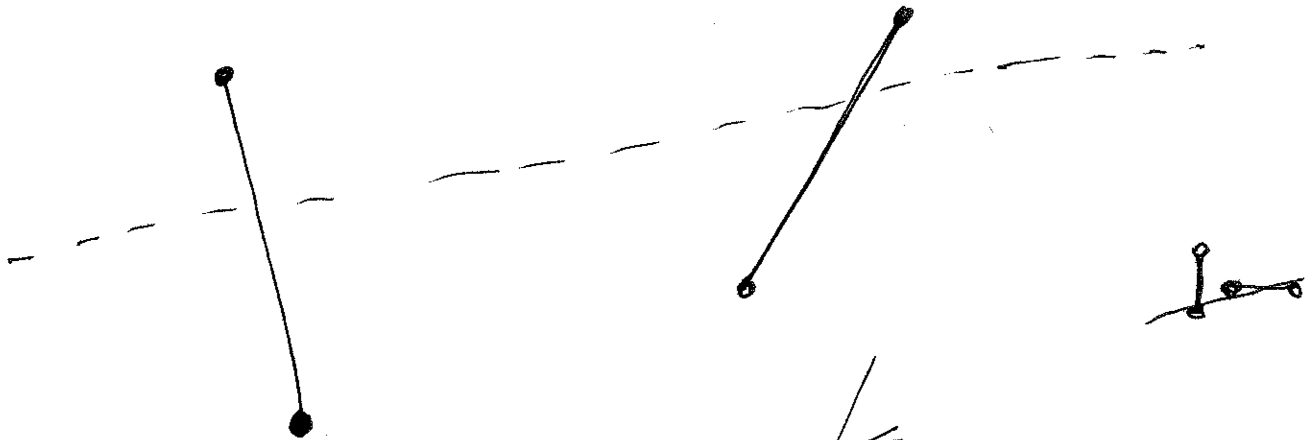


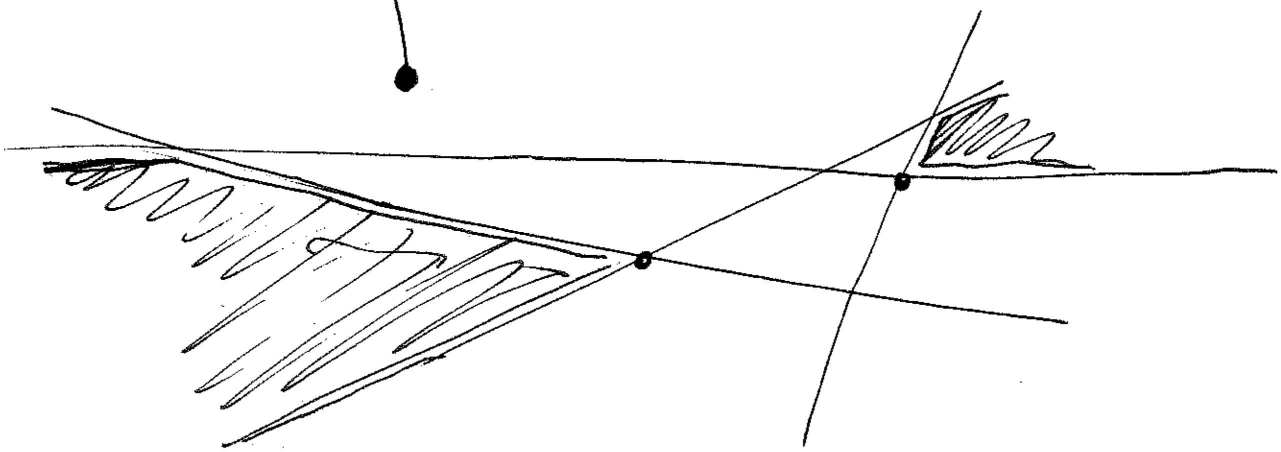
Note: 2 convex pieces.

Can it be only 1?

Can it be bounded?

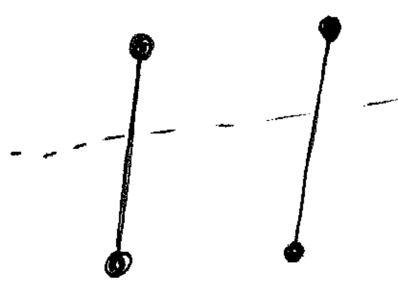
- What if no vertical line ~~[illegible]~~ separates P and Q? Is it convex?

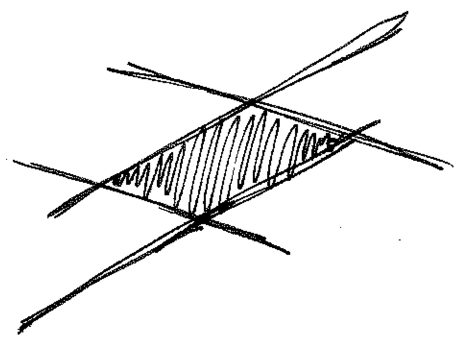




5.

All lines stabbing 2 vertical line segments





Exercise 6

Dual of all points on a line segment