

**Due: Before class on March 11, 2014**

## Homework 4

1. Did you do the reading? YES/NO/SORTA
2. Did you do the reading before class? YES/NO/SORTA
3. How long did you spend on this homework (rounding up)? \_\_\_\_\_ hours.

## 1 Context-Free Grammars

1.1 We have used the convention that uppercase letters are variables and lowercase letters are terminals, but that is not required. Consider the following grammar  $G_1$ .

$$\begin{aligned} q &\rightarrow a \mid ab \mid abc \mid A \mid AB \mid ABC \\ a &\rightarrow A \\ b &\rightarrow C \\ A &\rightarrow b \mid bb \mid AAq \mid c \end{aligned}$$

- What are the variables of  $G_1$ ?
- What are the terminals of  $G_1$ ?
- What is the start variable?

1.2 Consider the following grammar  $G_2$ .

$$\begin{aligned} A &\rightarrow BB \mid C \\ B &\rightarrow xB \mid Bx \mid y \\ C &\rightarrow xxCx \end{aligned}$$

Answer the following questions. Don't forget that there is a difference between  $\Rightarrow$  and  $\overset{*}{\Rightarrow}$ .

- What are the variables of  $G_2$ ?
- What are the terminals of  $G_2$ ?
- What is the start variable?
- True or False:  $B \overset{*}{\Rightarrow} BB$ ?
- True or False:  $xBx \Rightarrow xxBx$ ?
- True or False:  $xBx \Rightarrow xxBxx$ ?
- True or False:  $C \overset{*}{\Rightarrow} xxxxxCxxx$ ?
- True or False:  $A \overset{*}{\Rightarrow} xxxxyxx$ ?
- True or False:  $A \overset{*}{\Rightarrow} xxxxyxy$ ?
- Give an informal description of  $L(G_2)$ .

## 2 Ambiguous Grammars

**2.1** Show that the following grammar is ambiguous over the alphabet of parentheses  $\Sigma = \{(, )\}$ . Then, give an unambiguous grammar that recognizes the same language.

$$\begin{aligned}A &\rightarrow (A) \mid (A)(A) \mid B \\B &\rightarrow BB \mid AA \mid CC \\C &\rightarrow ABA \mid \varepsilon\end{aligned}$$

## 3 Pushdown Automata

**3.1** Give an informal description and a state diagram for a PDA that recognizes the language of all palindromes over the alphabet  $\{0, 1\}$ . Recall that a palindrome is a string that is the same forwards and backwards. Some examples include 0, 010, and 00111100.

**3.2** Let  $A$  be a regular language recognized by an NFA  $M = (Q_M, \Sigma, \delta_M, q_M, F_M)$  and let  $B$  be a context-free language recognized by a PDA  $P = (Q_P, \Sigma, \Gamma, \Delta_P, q_P, F_P)$ . Prove that  $A \cap B$  is context-free by constructing a PDA using  $M$  and  $P$  that recognizes  $A \cap B$ . **To think about:** Why might the same construction not work for two CFLs?