Due: Before class on March 25, 2014

Homework 5

1. Did you do the reading? YES/NO/SORTA

2. Did you do the reading before class? YES/NO/SORTA

3. How long did you spend on this homework (rounding up)? _____hours.

1 The Pumping Lemma for CFLs

1 Let A and B be languages over $\Sigma = \{0, 1\}$ defined as

$$A = \{(0 \cup 1)^a (1 \cup 2)^b (2 \cup 3)^c \mid a \geq b\}, \text{ and}$$
$$B = \{(0 \cup 1)^a (1 \cup 2)^b (2 \cup 3)^c \mid a = c\}.$$

- Prove that A and B are context-free.

- Prove that $A \cup B$ is context free. **Think about this:** Would your proof work for any pair of CFLs?

- Prove that $A \cap B$ is not context free. This implies that the context-free languages are not closed under intersection.

- It is a basic fact in set theory that $\overline{X \cup Y} = \overline{X} \cap \overline{Y}$. Use the previous problems to show that the class of conext-free languages are not closed under the complement operation.

2 Turing Machines

2 Give Turing Machines for the languages A, B, $A \cup B$, and $A \cap B$ from question 1. Indicate the transition functions by drawing the state diagram.