CSE3502 - Theory of Computation April 21, 2014

Not Due: Solutions will be posted next week

Homework 8

1. Did you do the reading? YES/NO/SORTA
2. Did you do the reading before class? YES/NO/SORTA
3. How long did you spend on this homework (rounding up)? _____hours.

1 Sanity Check

Prove that every language in NP is decidable. *(This is a test that you know the definitions)*

2 The Return of Graph Isomorphism

Recall the definition of graph isomorphism from homework 1. Prove that the following language is in NP.

$$GRAPHISOMORPH = \{\langle G, H\rangle \mid G \text{ and } H \text{ are isomorphic}\}$$

Interestingly, this language is not known to be either in P or in NP-COMPLETE.

3 Maybe we should bring back Roman Numerals.

The Partition problem is defined as

$$PARTITION = \left\{ \langle a_1, \ldots, a_k\rangle \mid \text{there exists a subset } B \subset \{a_1, \ldots, a_k\} \text{ such that } 2\sum_{b\in B} b = \sum_{i=1}^{k} a_i \right\}$$

Roman numerals use the alphabet $\{I, V, X, L, C, D, M\}$ to represent numbers, where

$$I = 1$$
$$V = 5$$
$$X = 10$$
$$L = 50$$
$$C = 100$$
$$D = 500$$
$$M = 1000$$

A number is represented as a string of these letters and the value of the number is the sum of the values of the symbols with some exceptions. If a symbol appears before a symbol of larger value, it is treated as negative. Thus, $IX = 9$, $XL = 40$, and $CM = 900$. There are some other rules, you may be familiar with, such as no symbol (other than M) can be repeated more than 3 times in a row. So, 4 must be written as IV rather than $IIII$. Other than some isolated negative symbols, the string symbols are sorted in decreasing order of value.

Now, $PARTITION$ is known to be NP-COMPLETE when numbers are written in binary. Prove that $PARTITION \in P$ if the input numbers are represented in Roman numerals. *Hint: Use dynamic programming.* Is it really more efficient to work with Roman numerals? What's going on here?